

Update on $K_{L,S} \rightarrow \pi^+ \pi^- \gamma$

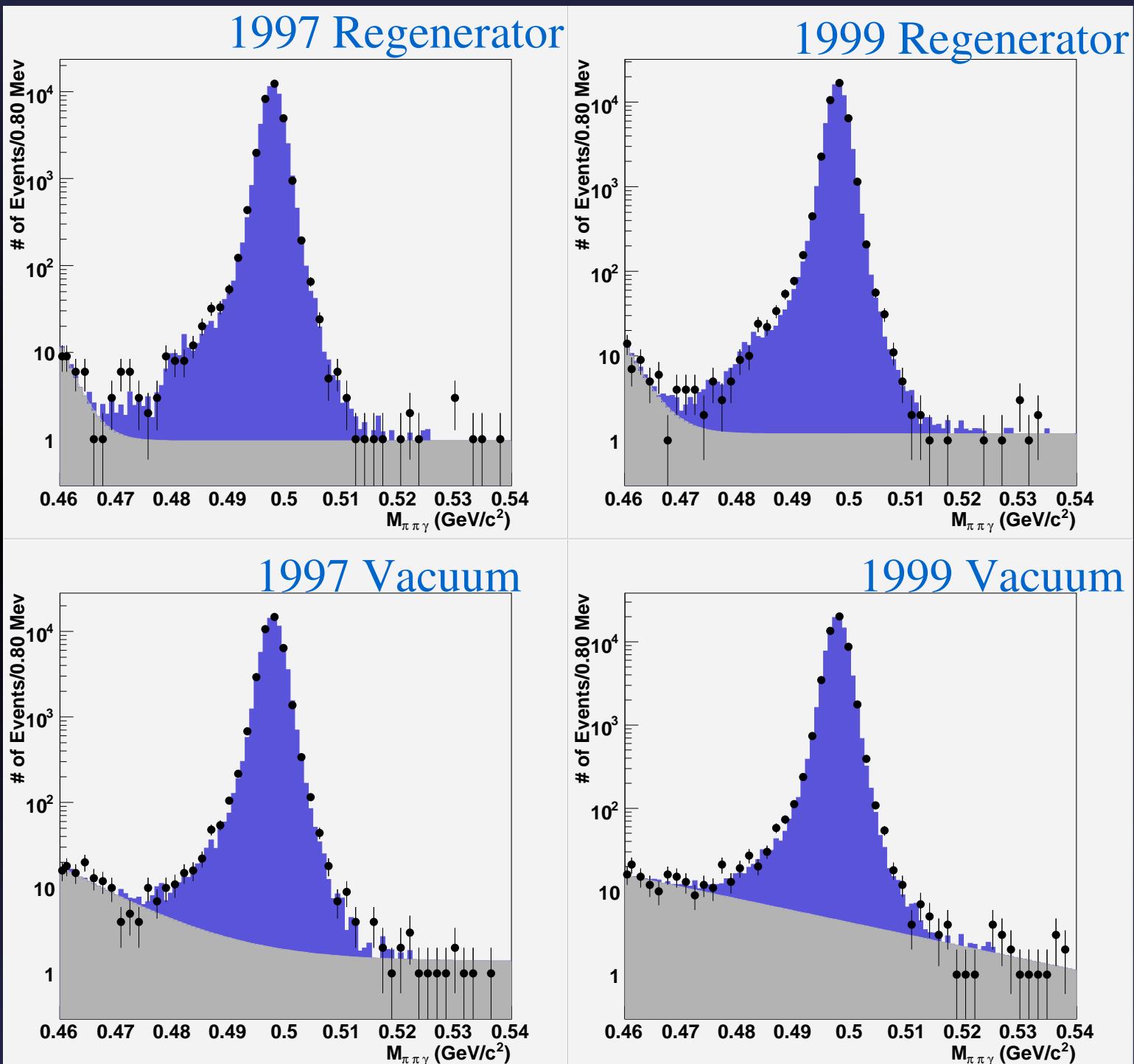
Michael Ronquest
KTeV meeting
November 17th 2007

Outline For Today's Talk

- Review of data sample
- Review of analysis
- Fit results
- Summary of systematic errors
- Final Result
- Computation of $\eta_{+-\gamma}$

Data sample

- After all cuts there are ~307,000 events in the total sample
- See list of cuts in back of talk
- <244 background events (0.08%)
- 40% $K \rightarrow \pi \mu \nu$
- 30% $K \rightarrow \pi e \nu$
- 30% $K \rightarrow \pi^+ \pi^- \pi^0$



Review

- Looking for E1 direct photon emission in $K_{L,S} \rightarrow \pi^+ \pi^- \gamma$
 - Two terms:
 - One violates CP directly
 - The other violates CP indirectly (in the K_L)
 - Direct CPV term only contributes to K_L
 - Term is doubly CP violating for K_S
 - Indirect CPV term contributes to K_L and K_S
 - Look for extra time dependent interference in the decay rate which is also dependent on photon energy
 - Direct CPV parameter here is $\hat{\epsilon}$

Decay Amplitudes

$$E_{IB}(K_S) = \left| 4 \frac{M_K^2}{E_\gamma^2} \right| \frac{e^{i\delta_0}}{1 - \beta^2 \cos^2(\theta)}$$

$$E_{IB}(K_L) = \left| 4 \frac{M_K^2}{E_\gamma^2} \right| \frac{\overbrace{\eta_{+-}}^{\epsilon + \epsilon'} e^{i\delta_0}}{1 - \beta^2 \cos^2(\theta)}$$

Fit for parameters in yellow

$$M(K_S) = i \epsilon g_{M1} \left| \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2 E_\gamma M_K} + 1 \right| e^{i\delta_1}$$

$$M(K_L) = i g_{M1} \left| \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2 E_\gamma M_K} + 1 \right| e^{i\delta_1}$$

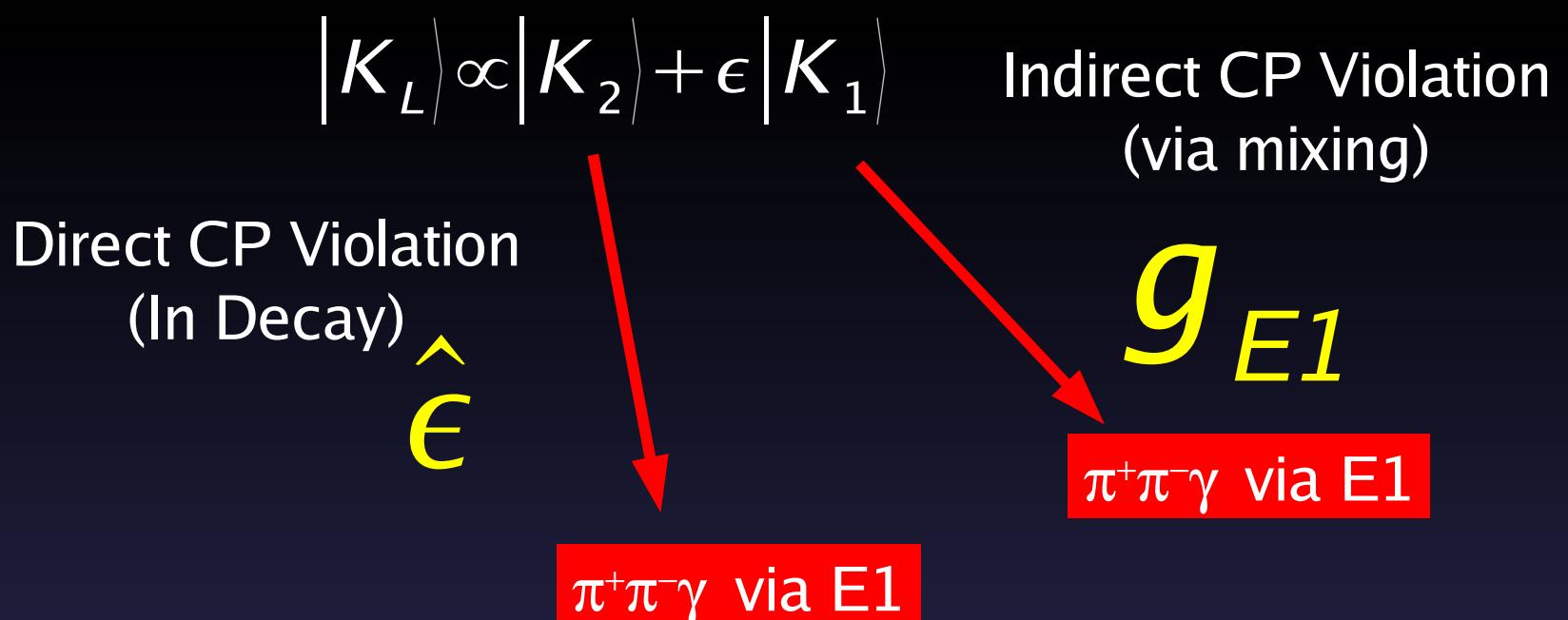
Direct CP violation parameter

$$E_{DE}(K_S) = \frac{g_{E1}}{\epsilon} e^{i(\delta_1 + \phi_\epsilon)}$$

$$E_{DE}(K_L) = \underbrace{g_{E1} e^{i(\delta_1 + \phi_\epsilon)}}_{\text{indirect CPV}} + \underbrace{i 16 \hat{\epsilon} e^{i\delta_1}}_{\text{direct CPV}}$$

Direct Vs Indirect CP Violation in E1

- The E1-DE K_L amplitude is a mixture of direct CP and indirect CP violating terms
- g_{E1} part of amplitude is present in K_L and K_S
- \hat{E} part is present in K_L only



Decay Rate for $K_{L,S} \rightarrow \pi^+ \pi^- \gamma$

- The decay rate is:

$$\frac{dN}{d\tau dE_\gamma d\cos(\theta)} = N_K \left[|\rho|^2 \left[\frac{d\Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}}{dE_\gamma d\cos(\theta)} e^{-\frac{\tau}{\tau_s}} + \frac{d\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}}{dE_\gamma d\cos(\theta)} e^{-\frac{\tau}{\tau_L}} \right] + 2R \left[\rho \frac{d\gamma_{LS}^*}{dE_\gamma d\cos(\theta)} e^{i\Delta m_K \tau} \right] e^{-\left(\frac{1}{\tau_L} + \frac{1}{\tau_S}\right)\frac{\tau}{2}} \right]$$

where:

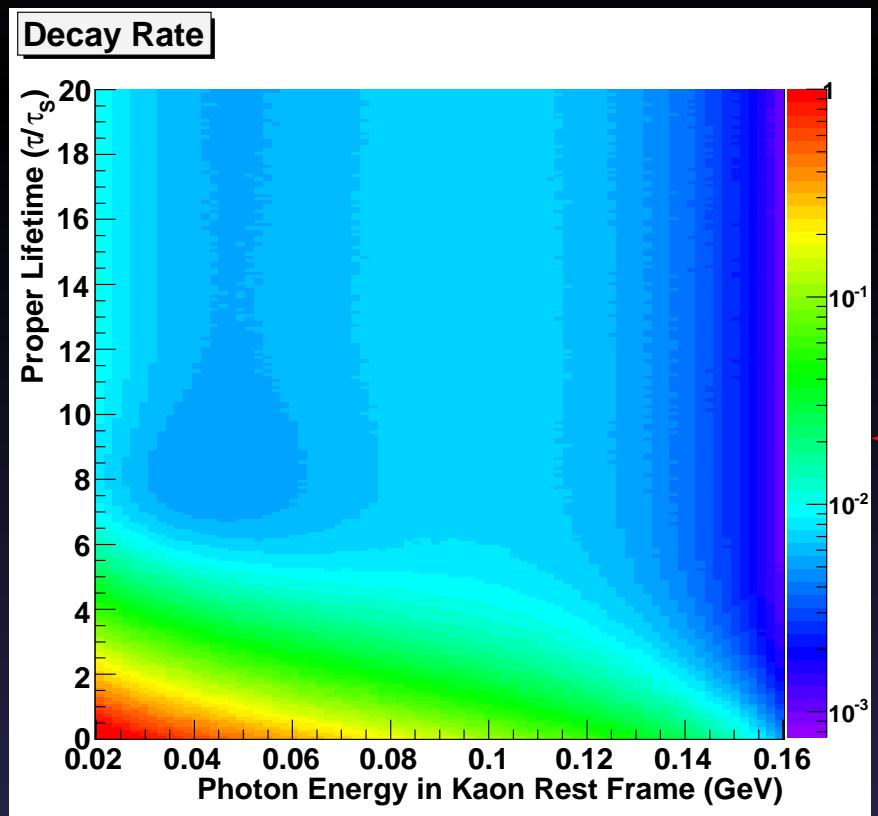
$$\frac{d\gamma_{LS}}{dE_\gamma d\cos(\theta)} \propto [E_{IB}(K_L) + E_{DE}(K_L)] * [E_{IB}^*(K_S) + E_{DE}^*(K_S)] + M(K_L)M^*(K_S)$$

$$\frac{d\Gamma_{K_L \rightarrow \pi^+ \pi^- \gamma}}{dE_\gamma d\cos(\theta)} \propto |E_{IB}(K_L) + E_{DE}(K_L)|^2 + |M(K_L)|^2$$

$$\frac{d\Gamma_{K_S \rightarrow \pi^+ \pi^- \gamma}}{dE_\gamma d\cos(\theta)} \propto |E_{IB}(K_S) + E_{DE}(K_S)|^2$$

Projections of Decay Rate

- The decay rate will give the density of events in phase space (τ , E_γ , $\cos\theta$)
- Plot of photon energy versus proper lifetime is interesting:



Decay Rate For
Regenerator Beam

Note dominance of inner brem
from K_S

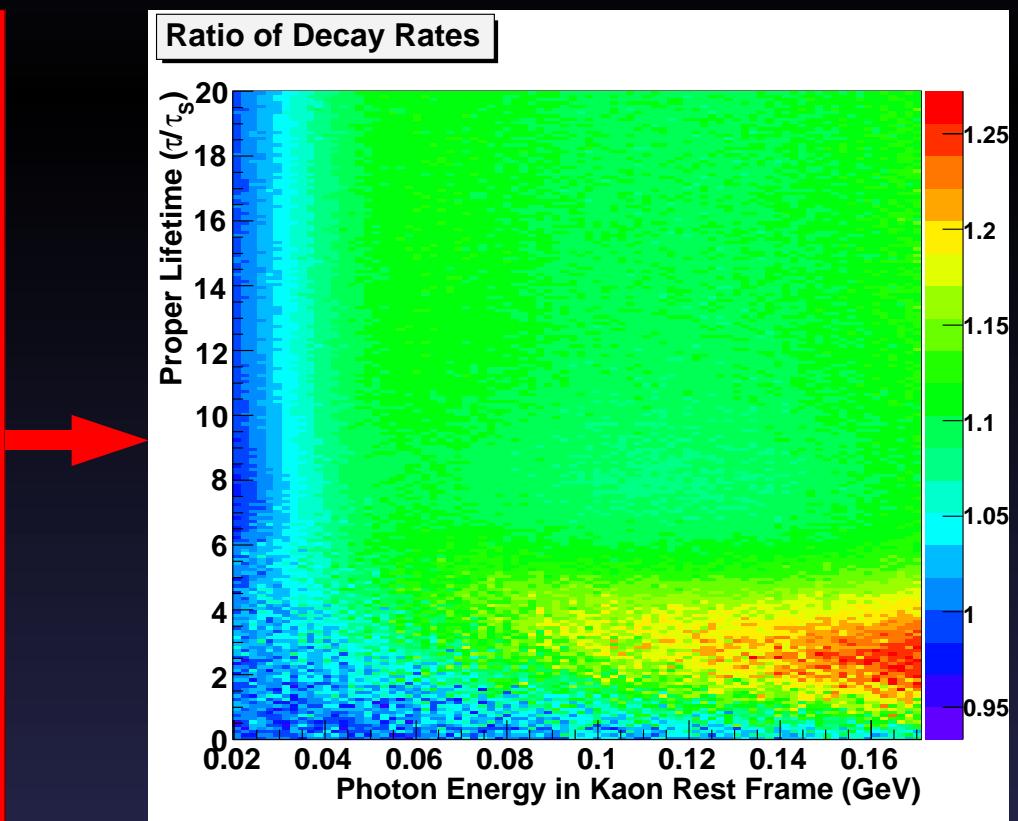
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Projections of Decay Rate

- The decay rate will give the density of events in phase space (τ , E_γ , $\cos\theta$)
- Plot of photon energy versus proper lifetime is interesting:

Ratio plot of decay rates (modified/nominal)

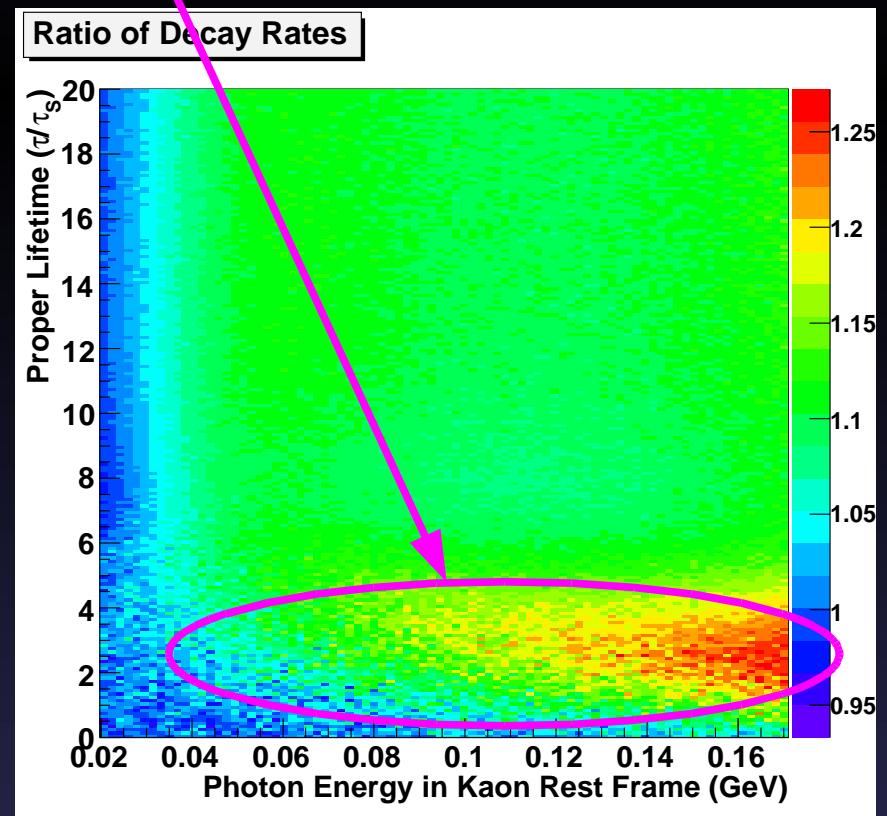
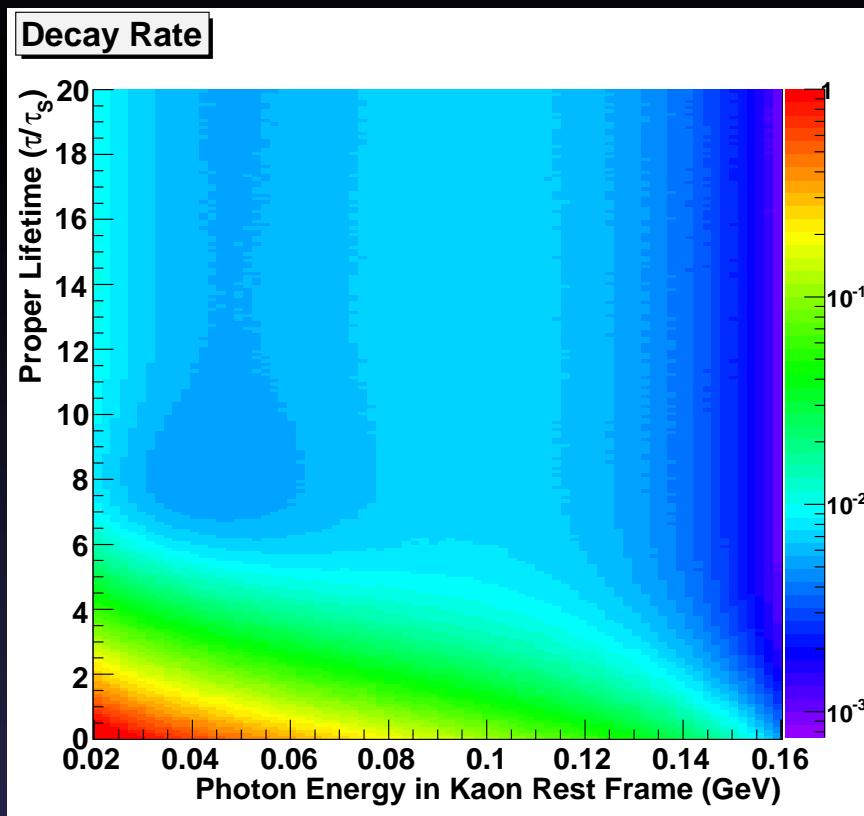
nominal = no direct CP term



Projections of Decay Rate

- Set $\hat{\epsilon}=0.01$, and then take the ratio of a plot with E1 DE and one without
- What happens if direct emission is present?

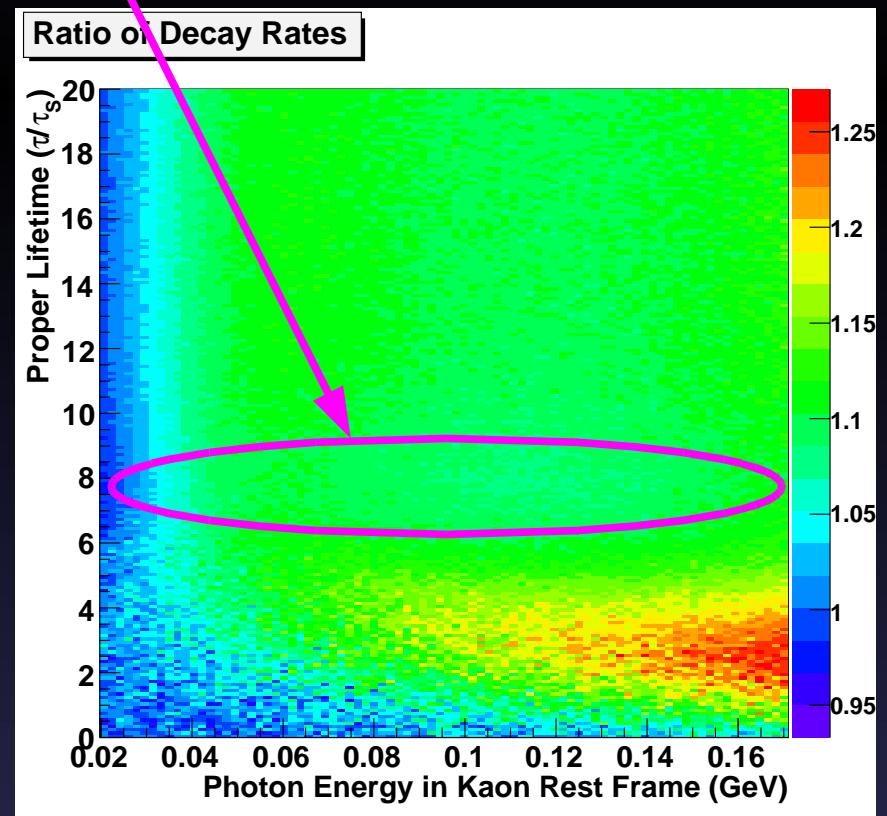
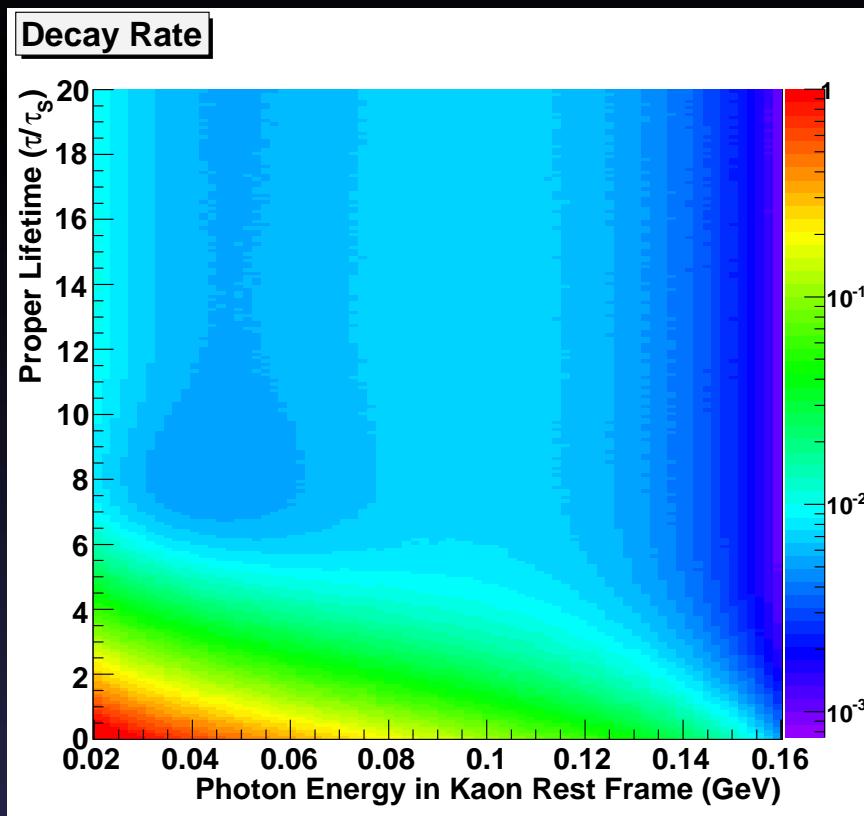
Constructive Interference Region



Projections of Decay Rate

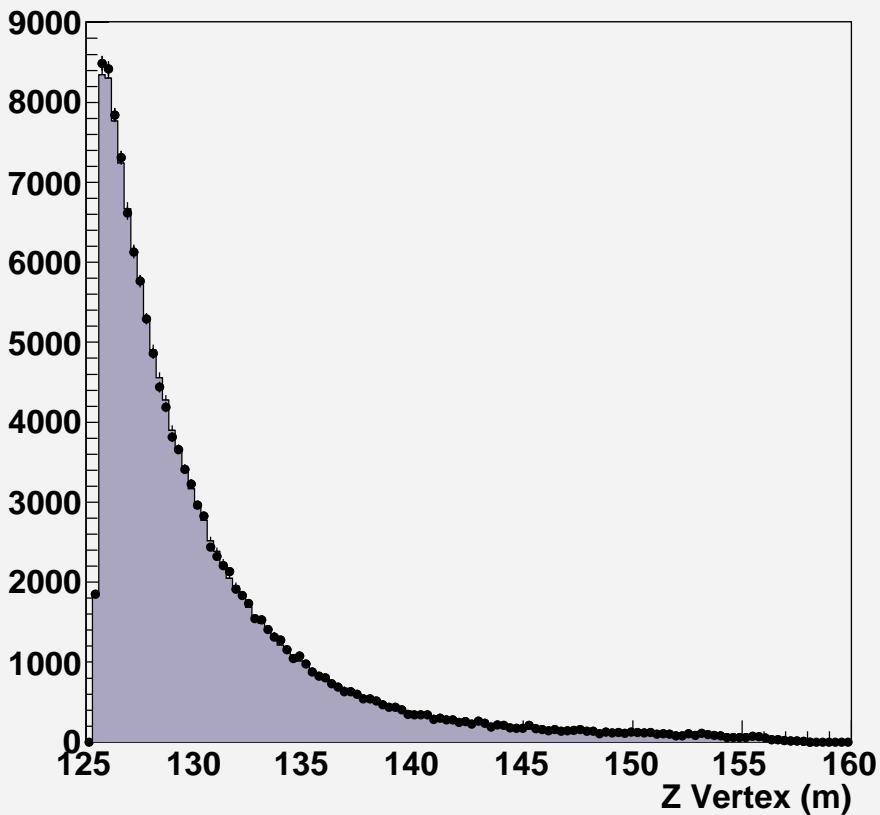
- Set $\hat{\epsilon}=0.01$, and then take the ratio of a plot with DE and one without
- What happens if direct emission is present?

Destructive Interference Region



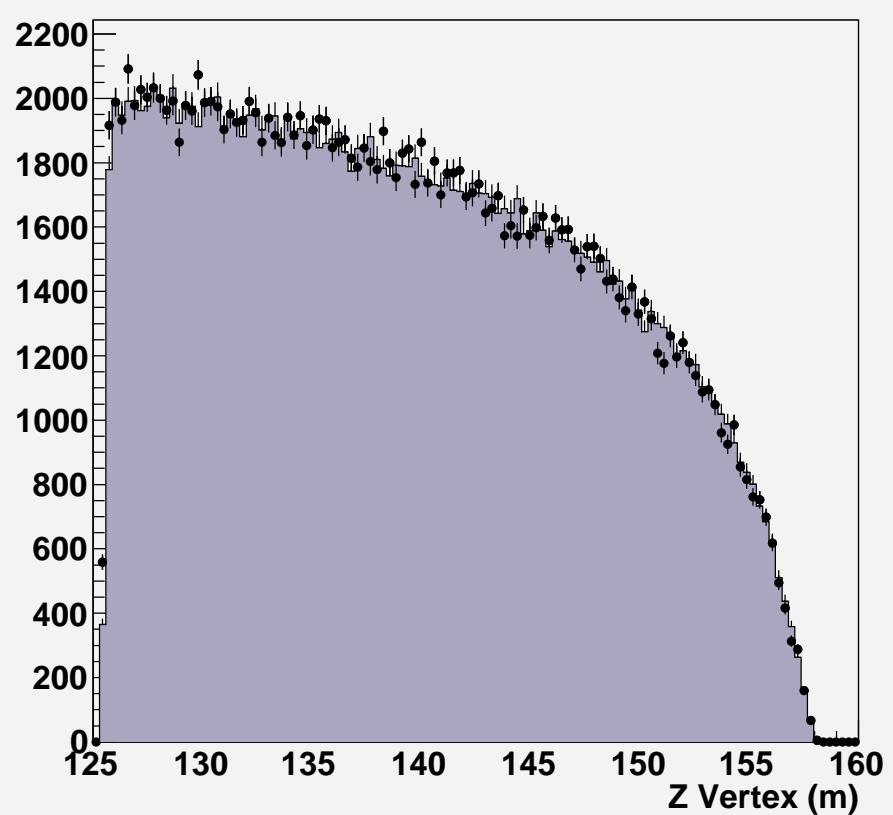
Z Location of Decay Vertex

$\chi^2/\text{dof} = 102.72/113.0$



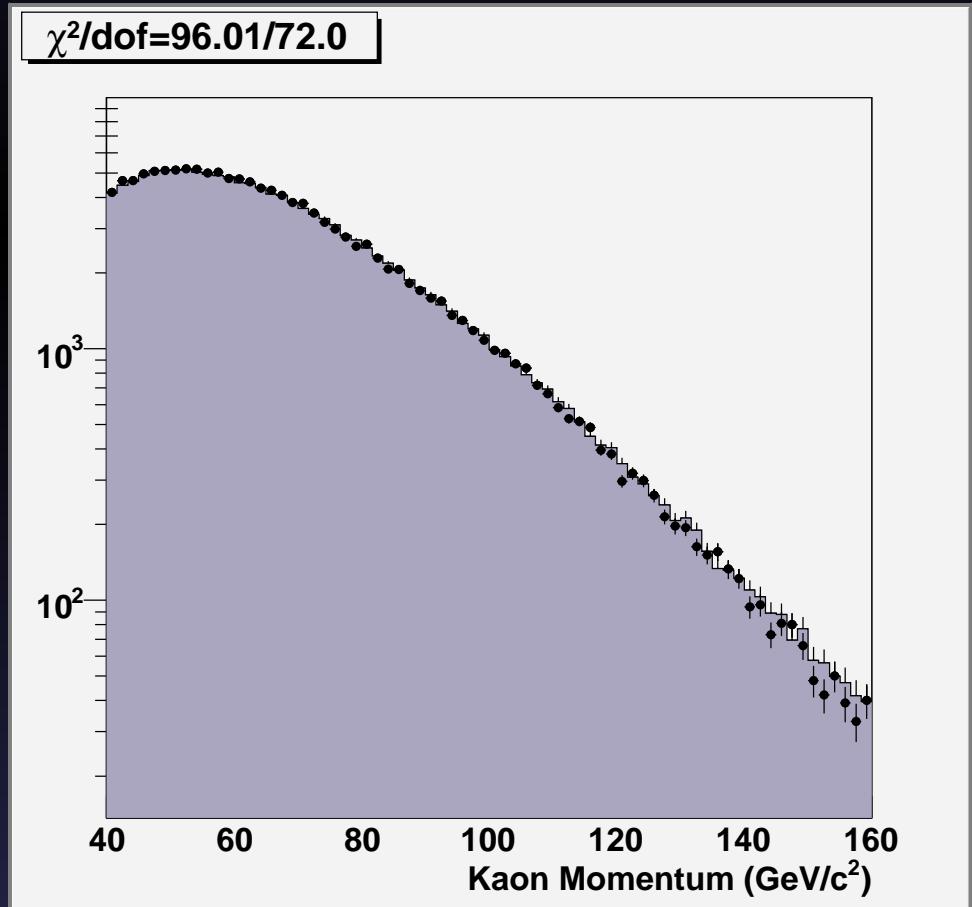
Regenerator Beam Data

$\chi^2/\text{dof} = 181.07/113.0$

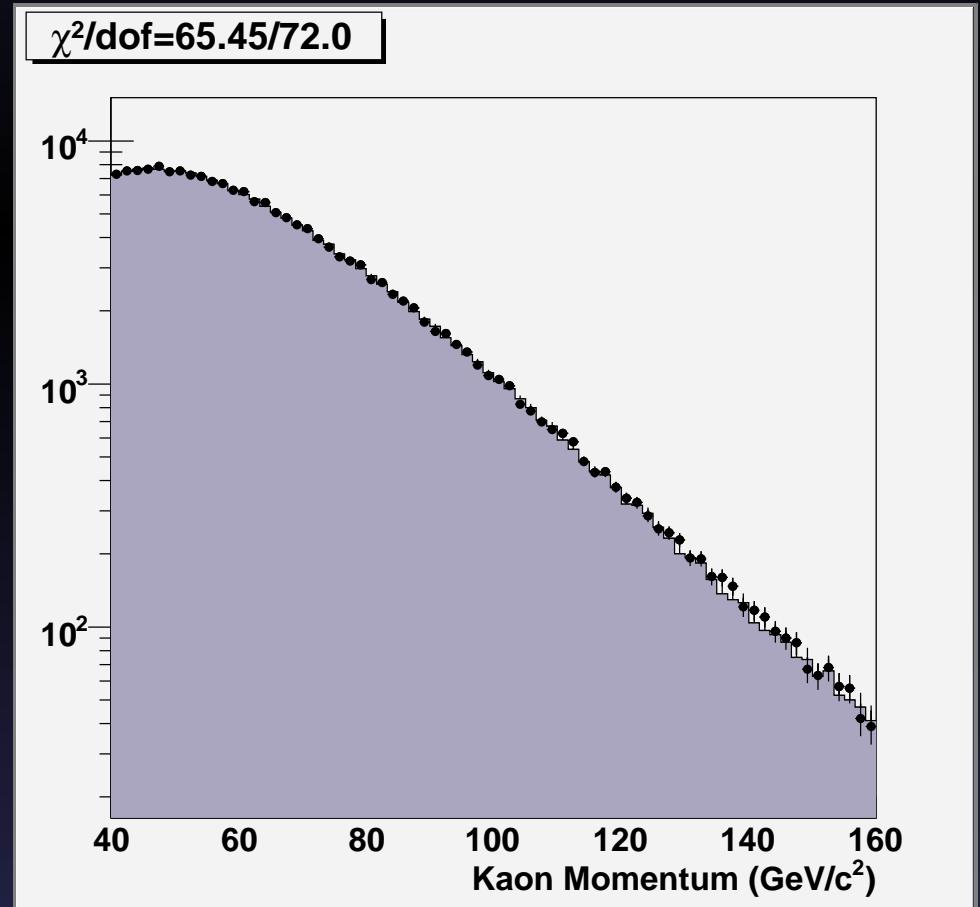


Vacuum Beam Data

Reconstructed Kaon Momentum



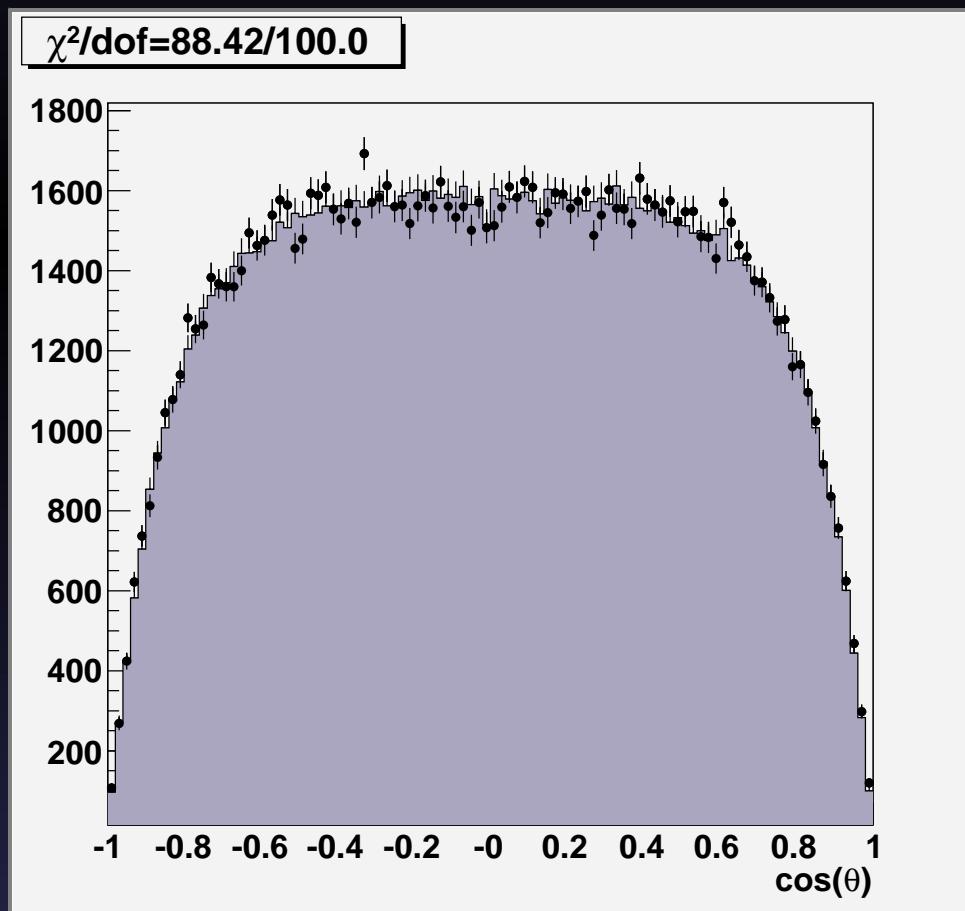
Regenerator Beam Data



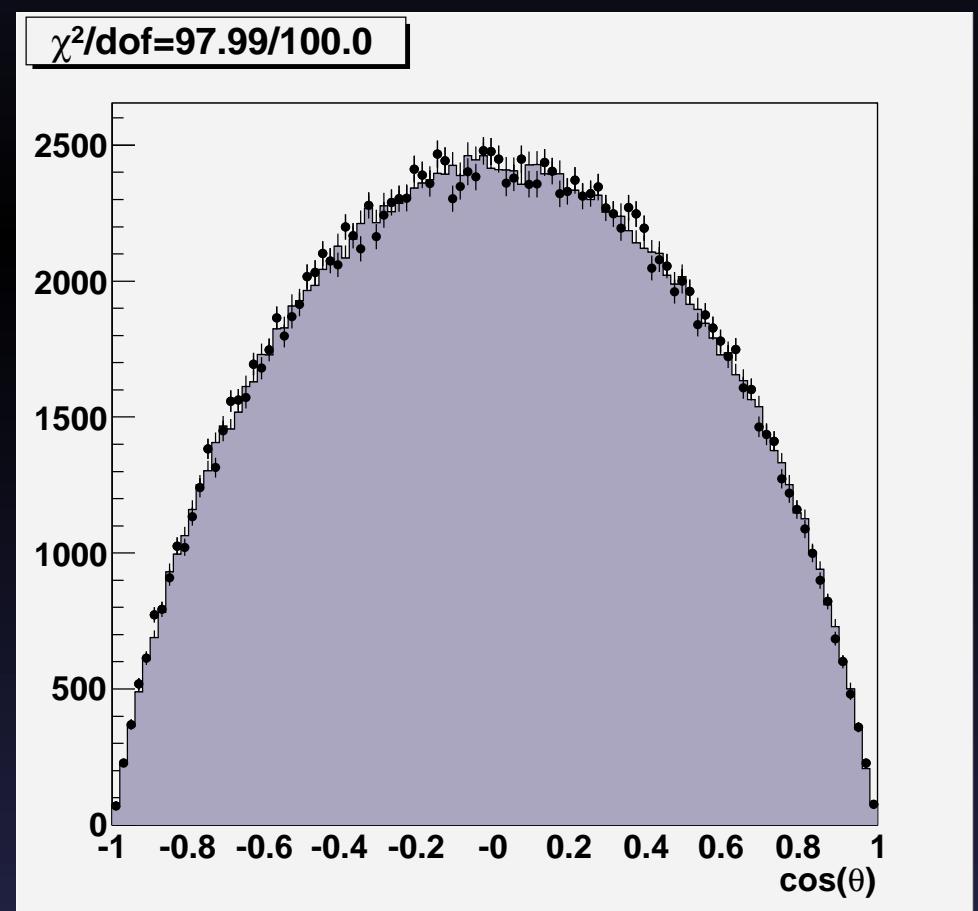
Vacuum Beam Data

Cosθ Of Photon

- Flat central portion at left due to Inner Bremsstrahlung dominance in K_S
- Rounded shape at right due to M1 direct emission in K_L



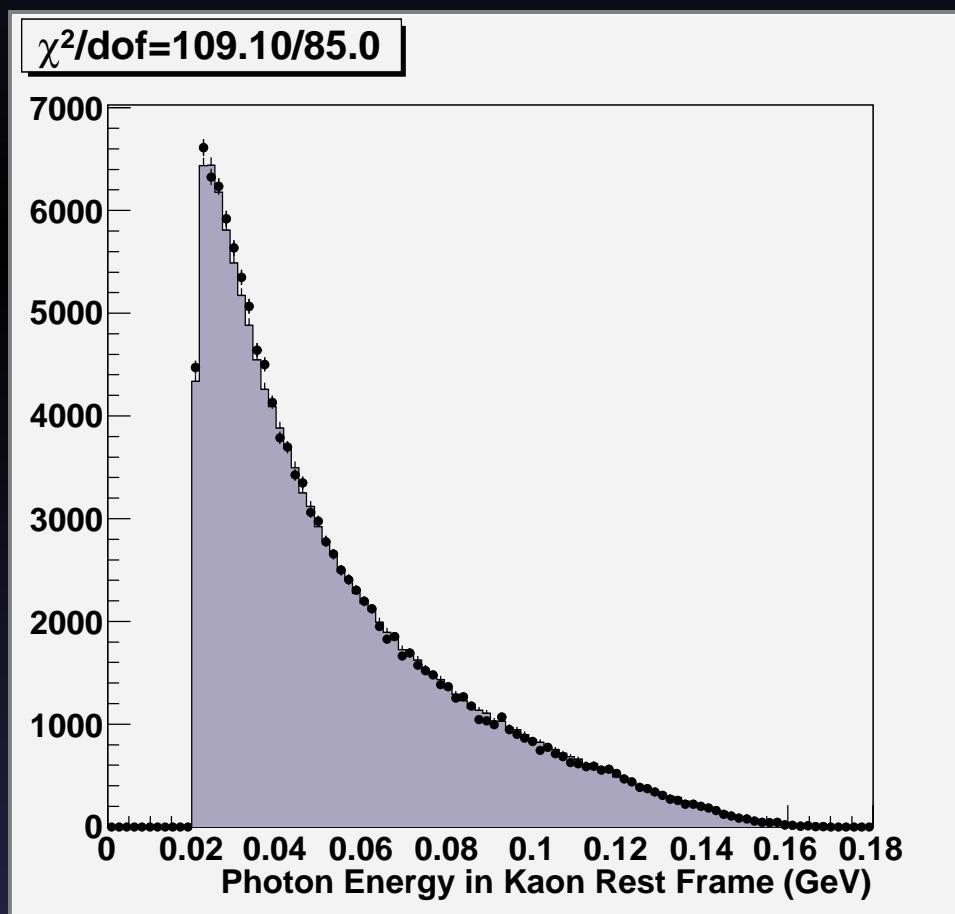
Regenerator Beam Data



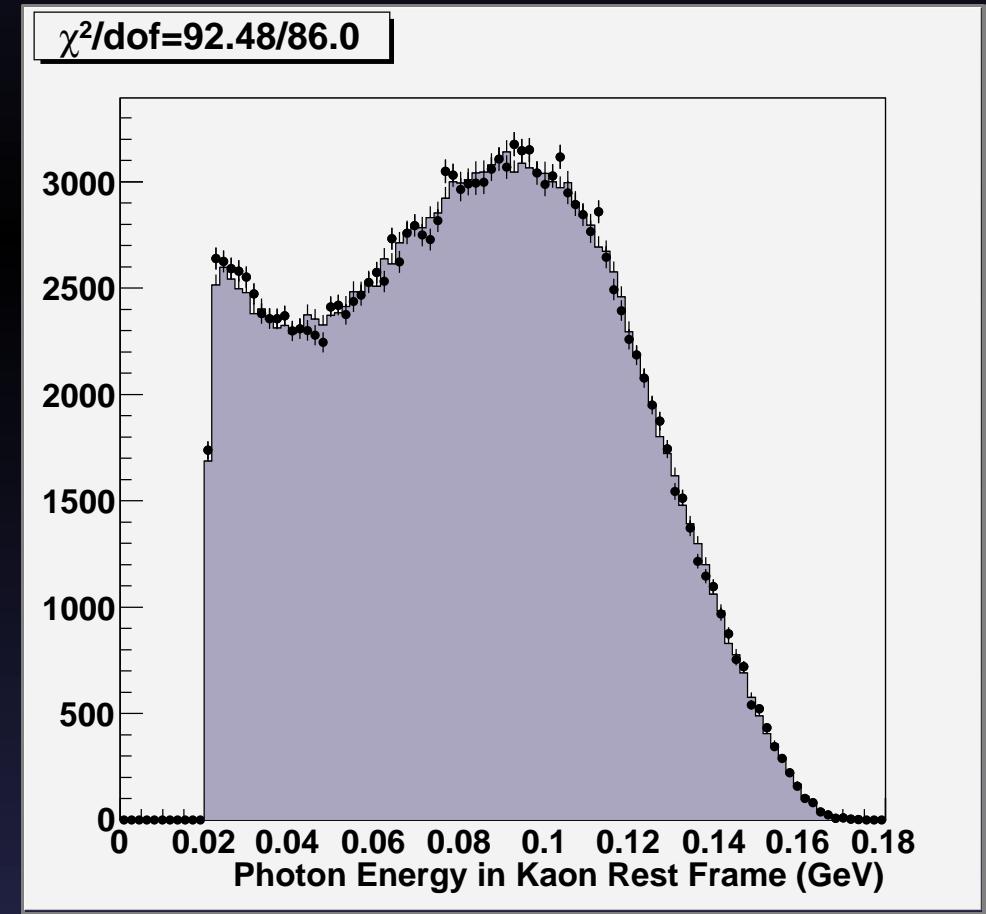
Vacuum Beam Data

E_γ in Kaon Rest Frame

- Clear Inner Brem dominance at left with very small M1 “hump” from the K_L component of regenerator beam
- CP conserving M1 Direct Emission overshadows smaller CP violating Inner Brem for K_L in vacuum beam



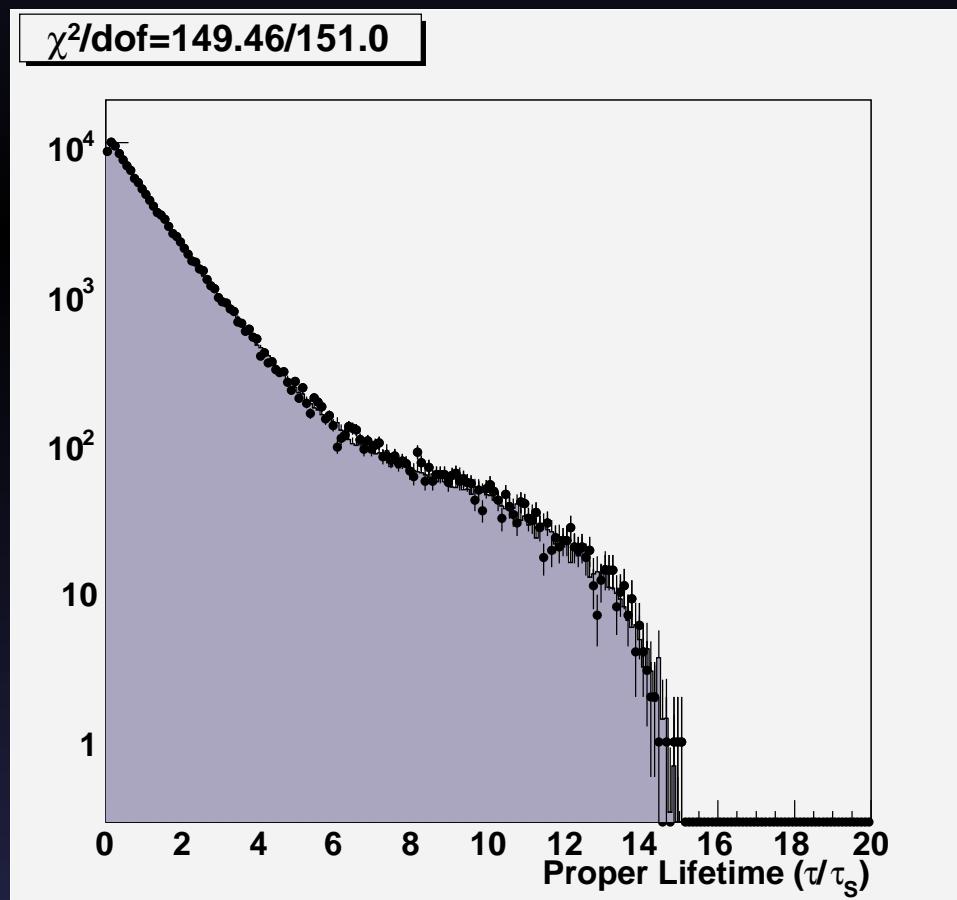
Regenerator Beam Data



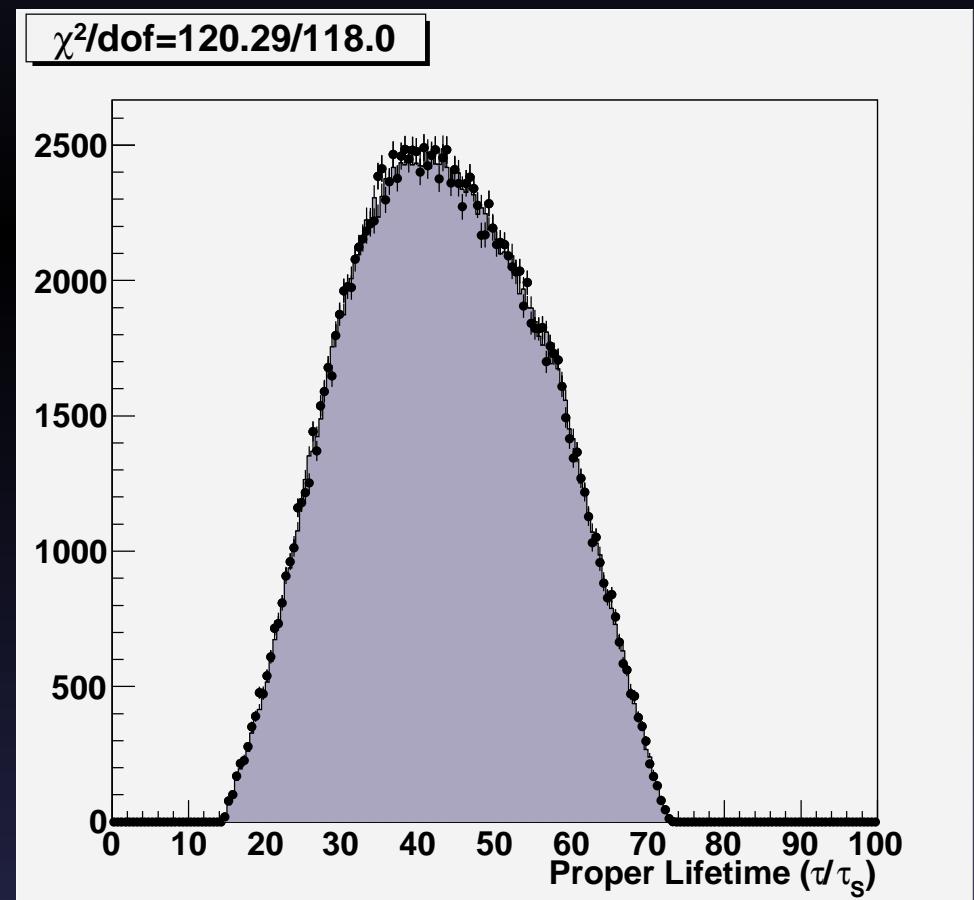
Vacuum Beam Data

Proper Kaon Lifetime

- Left plot shows kaon lifetime relative to end of regenerator. Both K_S and K_L components can be seen.
- Right plot shows kaon lifetime relative to target.



Regenerator Beam Data



Vacuum Beam Data

Likelihood fit

- The decay rate is used in a likelihood fit
 - Kaon wavefunctions and regenerator treated in the same way as epsilon prime fitter
 - Fit both years and both beams at the same time in a global fit
 - Vac beam helps constrain M1 parameters
 - Reg beam is sensitive to E1 parameters
 - Float M1 and E1 parameters
- Likelihood function is normalized using a large sample of reweighted MC events

Fitter Results

- Running the likelihood fit on the entire dataset we obtain:

- $\hat{\epsilon} = (3.87 \pm 0.65) \times 10^{-3}$
- $g_{E1} = (-6.1 \pm 1.5) \times 10^{-3}$
- $\tilde{g}_{M1} = 1.133 \pm 0.030$
- $a_1/a_2 = -0.750 \pm 0.007$

Errors and correlation matrix are computed using MINOS

- Correlations:

Small correlation !

Fit can distinguish between indirect and direct CP violating amplitudes!

	$\hat{\epsilon}$	g_{E1}	\tilde{g}_{M1}	a_1/a_2
$\hat{\epsilon}$	1	-0.367	-0.651	-0.527
g_{E1}	-0.367	1	0.327	0.267
\tilde{g}_{M1}	-0.651	0.327	1	0.983
a_1/a_2	-0.527	0.267	0.983	1

Systematic Errors

- Possible sources of systematic errors fall into three categories:
 - Incorrect calculation of decay rate
 - Occurs on a event by event basis
 - Examples: background, detector resolution
 - Incorrect normalization of likelihood function
 - Example: data/MC mismatch
 - Mixed effect
 - Example: input parameters used to compute decay rate for data and MC

Systematic due to input parameters

- There are a number of input parameters used in the likelihood fit
 - η_+ , regenerator parameters, kaon parameters, etc.
 - Vary each input within 1 sigma and observe the shift in fit parameters
 - Error=shift for uncorrelated inputs
 - Some input parameters are correlated
 - Must use the covariance matrix for the input parameters in order to combine the errors of the fit parameters

Systematic due to input parameters

- Largest effect comes from the combined error due to η_+ , α_{reg} and ρ
- Smaller contribution from ϕ_+ , Δ_M , τ_S
- Contribution from M_K , τ_L , regenerator transmission negligible

<i>Source of Error</i>	<i>Ehat error</i>	<i>GE1 error</i>	<i>GM1 error</i>	<i>a1/a2 error</i>
Uncertainty from Input Parameter Values	0.000346	0.000358	0.00746	0.000708

Systematic due to cuts

- Each cut could affect the fit
- Vary each cut and observe the shift “ s ” in fit parameters.
- Calculate error σ_s on shift using:

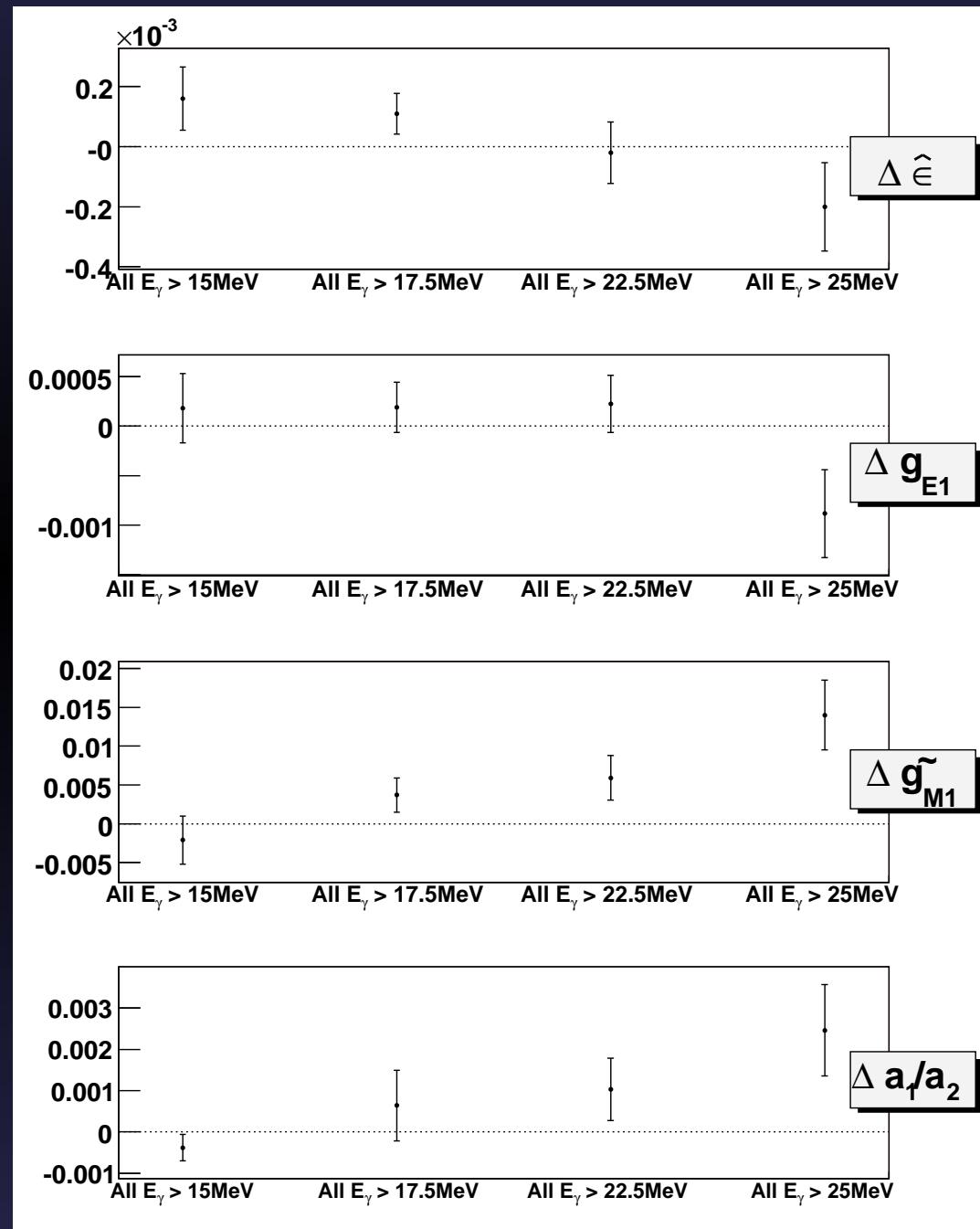
$$\sigma_s = \sqrt{\left| \sigma_{nominal}^2 - \sigma_{new}^2 \right|}$$

- Then compute error using Δ_s method:

$$\frac{1}{\sigma_s \sqrt{2\pi}} \int_{-\Delta_s}^{\Delta_s} dx \exp\left[\frac{(x-s)^2}{2\sigma_s^2} \right] = 0.683$$

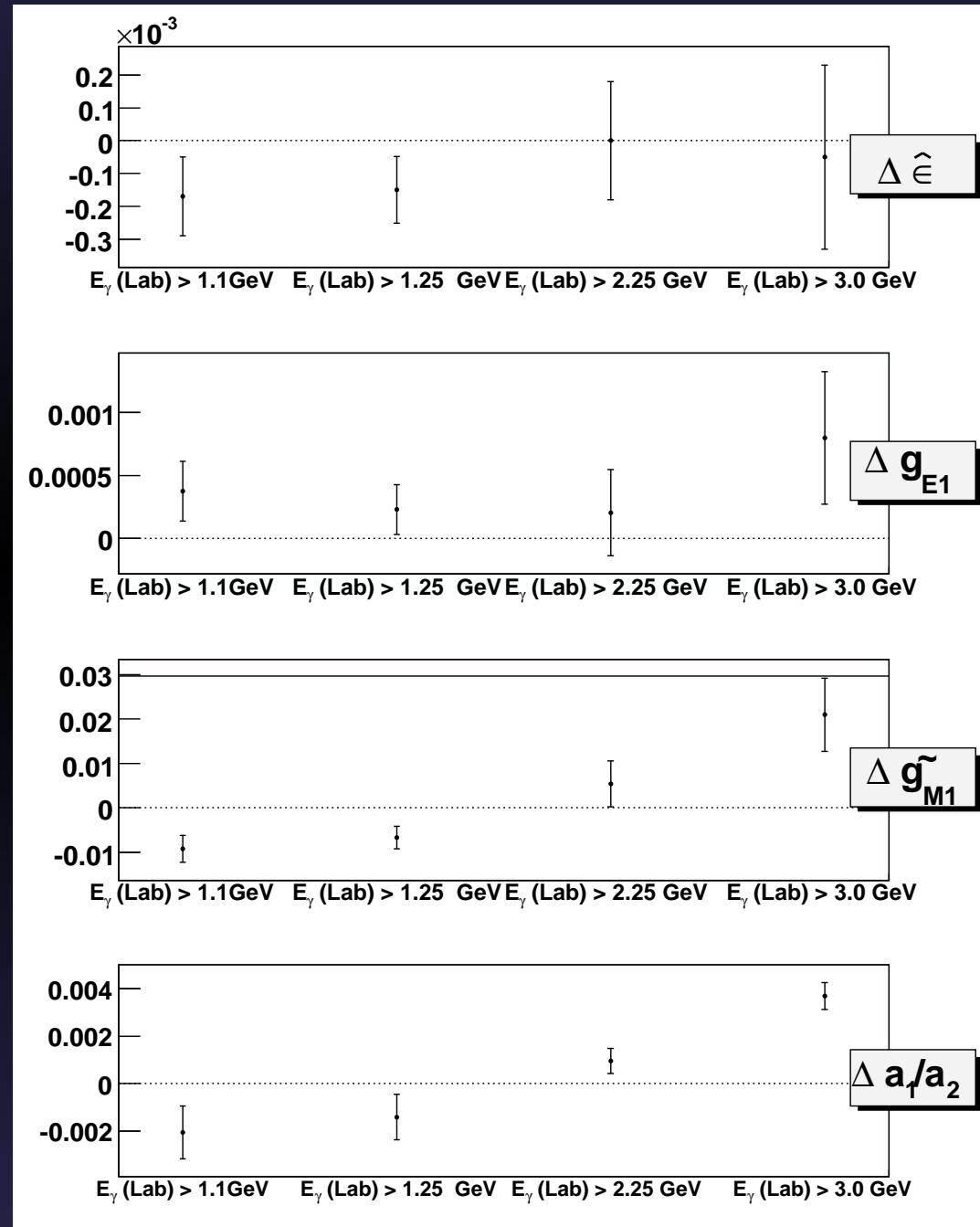
Systematic due to cuts

- Cut variation of E_γ (Kaon Rest frame)
- Large contribution to error on \hat{e}



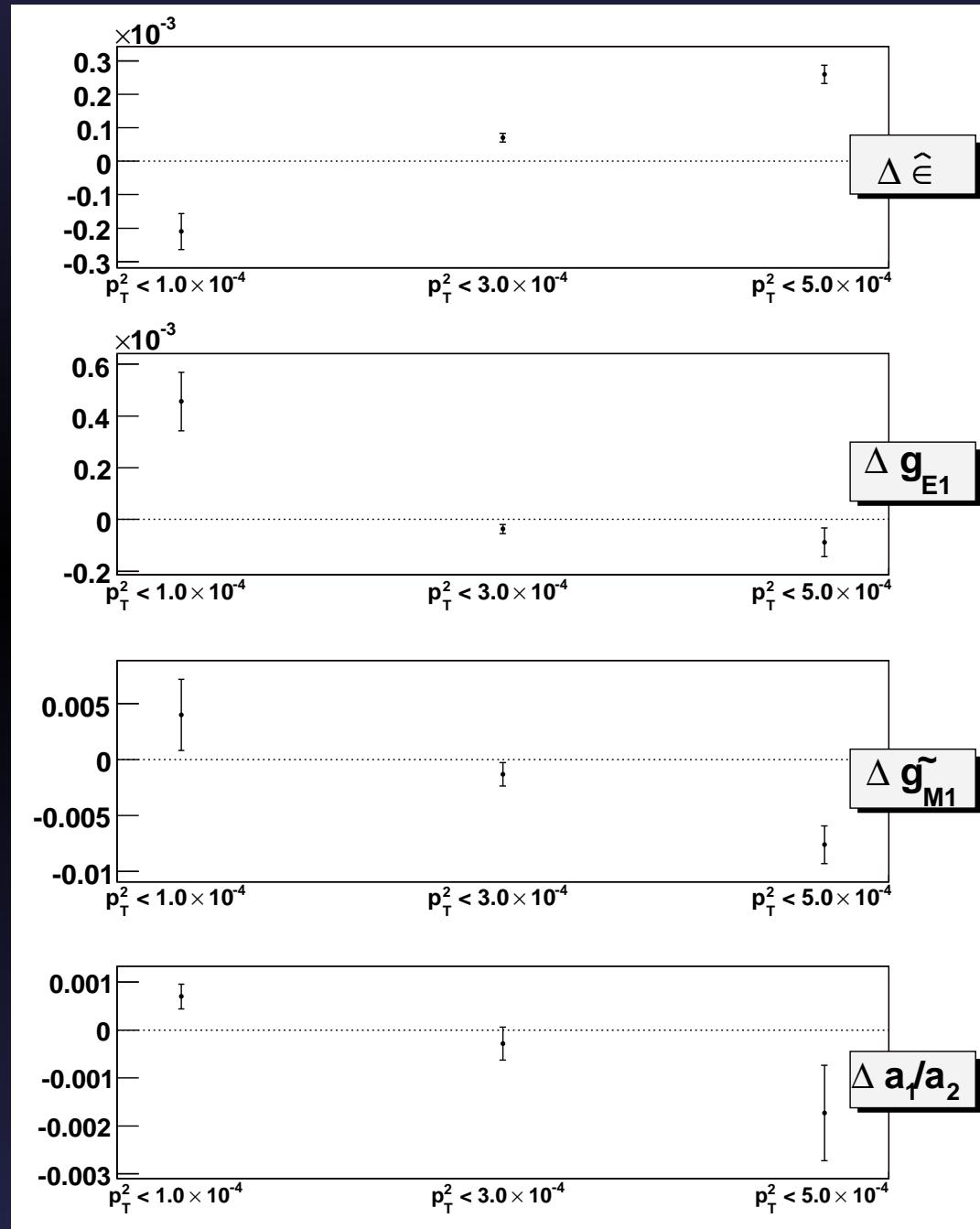
Systematic due to cuts

- Cut variation of E_γ (Lab frame)
- Large contribution to error on \hat{e}



Systematic due to cuts

- Cut variation of p_T^2
- Large contribution to error on ehat



Systematic due to cuts - summary

Varied Cut	\hat{e} Error	g_{E1} Error	$\widetilde{g_{M1}}$ Error	a_1/a_2 Error
No $\Lambda \rightarrow p\pi$ cut	0.0	0.0	0.0	0.0
No $\pi^0 \rightarrow \gamma\gamma$ mass cut	0.000127	0.00039	0.0081	0.00256
No Upstream track/ γ cut	0.0	0.000071	0.0	0.0
No Inner Ring Cut	0.000149	0.0	0.0	0.0
No Outer Ring Cut	0.000178	0.0	0.0	0.0
Track Y Separation	0.0	0.0	0.0	0.0
Track X Separation	0.00021	0.000468	0.00365	0.0
Track Separation	0.0	0.00053	0.0	0.0
P_π	0.000154	0.00131	0.007	0.0
E/p	0.000157	0.000463	0.0073	0.00126
$P_{\pi^0}^2$	0.000171	0.00036	0.0111	0.00306
$\pi - \gamma$ Separation	0.0	0.00131	0.0	0.0
p_T^2	0.000273	0.00051	0.0084	0.00221
All E_γ	0.00027	0.00109	0.0161	0.00299
E_γ (Lab)	0.000228	0.00105	0.0249	0.00396
χ^2_{OFFSET}	0.0	0.000351	0.0	0.0
χ^2_{VERTEX}	0.000118	0.000317	0.0	0.0
χ^2_{FUSION}	0.000083	0.000397	0.00106	0.0
Early Energy	0.0000269	0.0	0.0	0.0
Total Error	0.00064	0.00271	0.0355	0.0069

Source of Error	$Ehat$ error	$GE1$ error	$GM1$ error	$a1/a2$ error
Uncertainty from Input Parameter Values	0.000346	0.000358	0.00746	0.000708
Unceraintiy From Cut Values	0.000642	0.00271	0.0355	0.00685

Systematic due to PHOTOS

- Using PHOTOS “out of the box” to produce a second photon (i.e. $K \rightarrow \pi\pi\gamma\gamma$) doesn't produce great results
 - PHOTOS overestimates the size of $K \rightarrow \pi\pi\gamma\gamma$ according to the size of the low mass tail in the $\pi\pi\gamma$ invariant mass plot.
 - Using this region, scale PHOTOS so better agreement is reached. Error in scale will be a systematic.

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PHOTOS Uncertainty	0.00016	0.00101	0.0165	0.00537

Systematic due to accidentals

- The simulation of accidental activity is known to be accurate to roughly 10%
- Generate another normalization MC sample, this time with accidental simulation turned OFF.
- Refit, using this new MC sample to normalize the L.F.
- Error is 10% of the shift in parameters

<i>Source of Error</i>	<i>Ehat error</i>	<i>GE1 error</i>	<i>GM1 error</i>	<i>a1/a2 error</i>
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Uncertainty in Accidental Activity Sim	0.000027	0.00013061	0.00053	0.000157

Systematic due to resolution

- Detector resolution will be another systematic error. Sources will be:
 - Vertex resolution
 - Momentum resolution
 - Calorimeter resolution
- Resolution effects will smear values of E_γ , $\cos(\theta)$ and τ (via p_K and z_{vertex}) which are inputs into L.F.

Systematic due to resolution

- Use Monte Carlo data as “fake” data
 - We know the Monte Carlo truth for:
 - Photon energy, $\cos(\theta)$, kaon momentum and decay vertex
 - Fit the “fake” data using MC truth for everything
 - Refit using “reconstructed” values for everything
 - Difference in fit parameters yields estimated error due to resolutions

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Reconstruction Errors	0.00052	0.000325	0.0033	0.00009

Systematic due to resolution

- Note that the error estimate produced by this method also includes the systematic error introduced by:
 - Selection of the “wrong” photon cluster
 - Energy loss due to $K \rightarrow \pi\pi\gamma$
- In other words, the effect due to “bad” $K \rightarrow \pi\pi\gamma$ events is covered here

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Systematic due to background

- Overall method is to add additional background events to the nominal sample, and observe the shift in parameters
 - Can't subtract backgrounds in likelihood fits
 - Compare nominal sample with background to sample with double background
 - Sample changes, so we must use Δs prescription to estimate the systematic error

Systematic due to background

- First we need to determine the size of the background:
 - Fit the wings of the $\pi\pi\gamma$ mass distribution and extrapolate under the signal region
 - Do the same with the p_T^2 plot
 - For each sample (97 vs 99, reg vs vac), use the larger of the two numbers.

Systematic due to background

- Next, we need background events to add
 - Select events from the wings of the mass plot, well away from the kaon peak
 - Doing so ensures that we are picking up actual background decays
 - Using wings of p_T^2 plot would pick up scattered K- $\rightarrow\pi\pi\gamma$ events which will be dealt with later
 - The number of events to extract was already determined per the previous slide

Systematic due to background

- The extra background events are then added to the nominal sample, and this combined sample is then fit.
 - The difference in fit parameters is then plugged into the Δs prescription to obtain the systematic error

<i>Source of Error</i>	<i>Ehat error</i>	<i>GE1 error</i>	<i>GM1 error</i>	<i>a1/a2 error</i>
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Reconstruction Errors	0.00052	0.000325	0.0033	0.00009
Background	0.000185	0.0	0.00603	0.00177

Systematic due to incoherent regeneration

- The fitting program assumes each reg event is a result of coherent regeneration
- Incoherent regeneration produces a different kaon state than coherent, and will introduce a systematic error
 - Collimator scatters also have a different state, but the size of this background is negligible

Systematic due to incoherent regeneration

- MC includes simulation of incoherent regeneration, with proper normalization.
- Generate an MC sample with incoherent regeneration turned on
- Treat this MC sample as fake data, and fit
- Strip off incoherent events and refit
- Use Δs prescription to estimate the systematic error from the fit parameter shifts

Systematic due to incoherent regeneration

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Reconstruction Errors	0.00052	0.000325	0.0033	0.00009
Background	0.000185	0.00	0.00603	0.00177
Incoherent Regeneration	0.000123	0.000109	0.00	0.00

Systematic due to data/MC mismatch

- There are slopes in the data/MC plots of kaon momentum and z vertex
 - This will introduce an systematic error due to improper normalization of the likelihood function
- Reweight the MC sample event by event in order to remove the slope, and then refit

Systematic due to data/MC mismatch

- No additional statistical error is introduced here, so take the shift in fit parameters as the systematic error.

<i>Source of Error</i>	<i>Ehat error</i>	<i>GE1 error</i>	<i>GM1 error</i>	<i>a1/a2 error</i>
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Reconstruction Errors	0.00052	0.000325	0.0033	0.00009
Background	0.000185	0.00	0.00603	0.00177
Incoherent Regeneration	0.000123	0.000109	0.00	0.00
Flattened Distributions	0.00052	0.00136	0.0032	0.00062

Total Systematic Errors

- Total systematic errors are:

<i>Source of Error</i>	<i>Ehat error</i>	<i>GE1 error</i>	<i>GM1 error</i>	<i>a1/a2 error</i>
Uncertainty from Input Parameter Values	0.000346	0.000358	0.00746	0.000708
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Reconstruction Errors	0.00052	0.000325	0.0033	0.00009
Background	0.000185	0.00	0.00603	0.00177
Incoherent Regeneration	0.000123	0.000109	0.00	0.00
Flattened Distributions	0.00052	0.00136	0.0032	0.00062
Total Error	0.00107	0.00324	0.0406	0.00893

Final Results

- The final results, after all systematics, are:

- $\hat{\varepsilon} = (3.87 \pm 0.65(\text{stat}) \pm 1.07(\text{syst})) \times 10^{-3}$
- $g_{E1} = (-6.1 \pm 1.5(\text{stat}) \pm 3.2(\text{syst})) \times 10^{-3}$
- $\tilde{g}_{M1} = 1.133 \pm 0.030(\text{stat}) \pm 0.041(\text{syst})$
- $a_1/a_2 = -0.750 \pm 0.007(\text{stat}) \pm 0.009(\text{syst})$

- g_{E1} is significant at 1.7σ
- $\hat{\varepsilon}$ is significant at 3.1σ !

Final Results – E1 Direct Emission

- Recall E1 amplitude for K_L is:

$$E_{DE}(K_L) = \underbrace{g_{E1} e^{i(\delta_1 + \phi_\epsilon)}}_{\text{Indirect CPV}} + \underbrace{i 16 \hat{\epsilon} e^{i\delta_1}}_{\text{Direct CPV}}$$

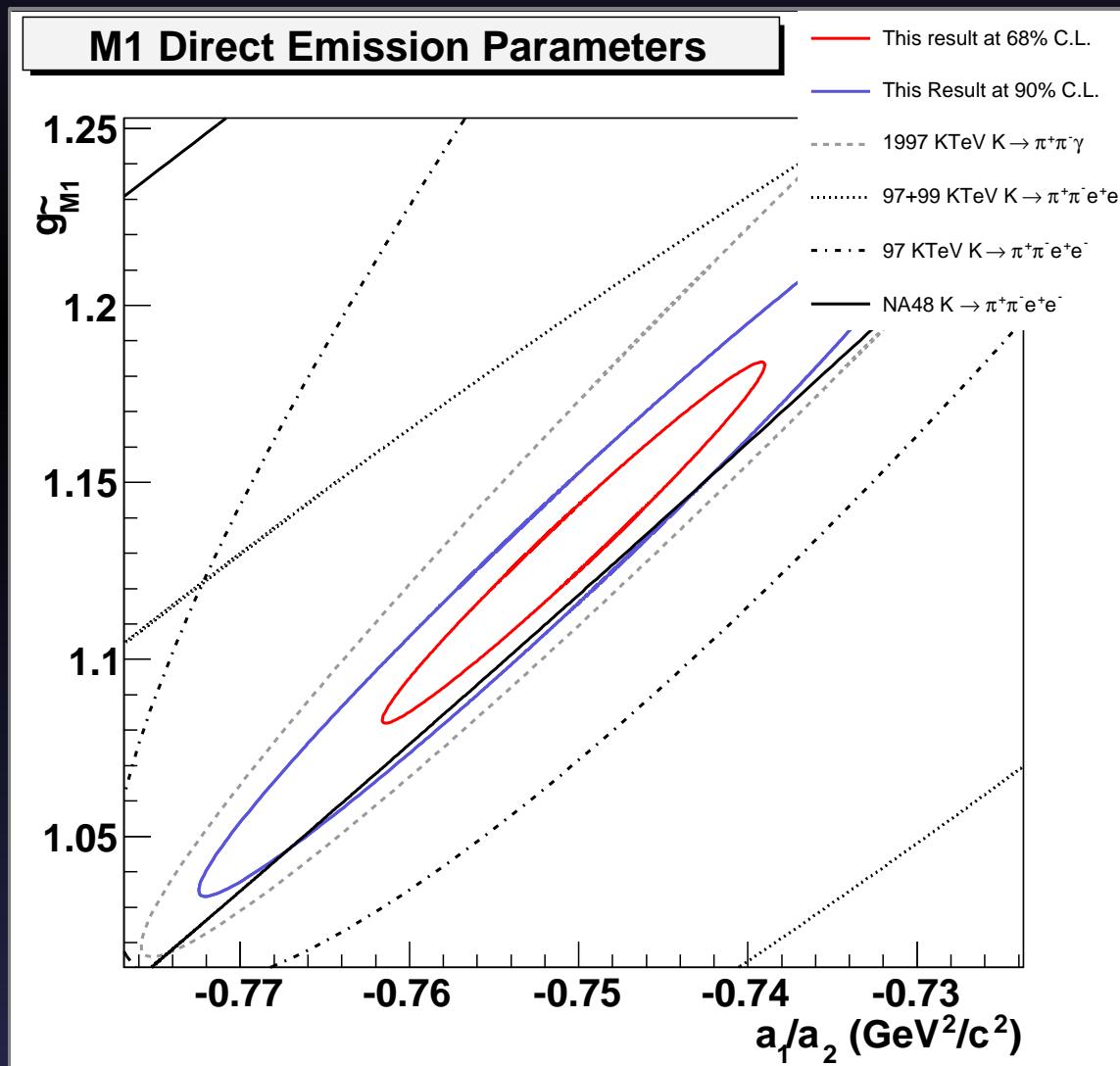
- Compare indirect and direct CP violating components:

$$F_{DCPV} = \frac{|16 \hat{\epsilon}|}{|16 \hat{\epsilon}| + |g_{E1}|}$$

- $F_{DCPV} = 91 \pm 4.5\%$ of the direct emission amplitude violates CP directly !?

Final Results – M1 Direct Emission

- M1 direct emission results are consistent with all other studies at 90% confidence



Final Results – M1 Direct Emission

<i>Study</i>	<i>g_{M1}</i>	<i>a1/a2</i>
This Study	1.133 +/- 0.030(stat) +/- 0.0406(syst)	-0.7503 ^{+0.0068} _{-0.0072} (stat) +/- 0.00893(syst)
KTeV 1997 $K_L \rightarrow \pi\pi\gamma$	1.198 +/- 0.035(stat) +/- 0.086(syst)	-0.738 +/- 0.007 (stat) +/- 0.018(syst)
KTeV 97+99 $K_L \rightarrow \pi\pi ee$	1.11 +/- 0.12(stat) +/- 0.08(syst)	-0.744 +/- 0.027 (stat) +/- 0.032 (syst)
NA48 $K_L \rightarrow \pi\pi ee$	0.99 +/- 0.3(stat) +/- 0.07(syst)	-0.81 ^{+0.07} _{-0.13} (stat) +/- 0.02 (syst)

Final Results - $\eta_{+-\gamma}$

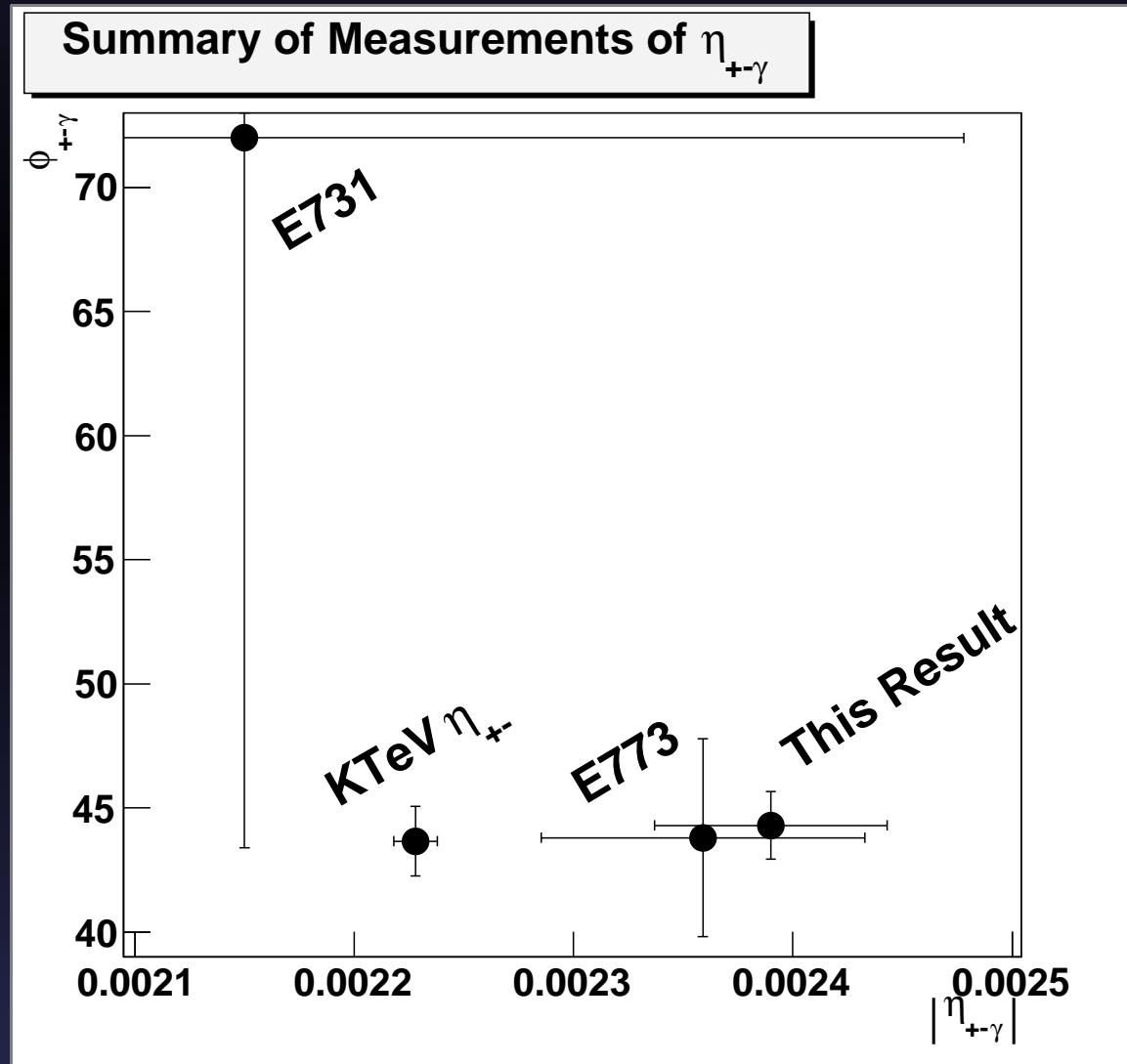
- We can use the fit results and compute $\eta_{+-\gamma}$:

$$\eta_{+-\gamma} = \eta_{+-} + \frac{\int d[PS] \tilde{\epsilon}'_{+-\gamma} |E_{IB}(K_S) + E_{DE}(K_S)|^2}{\int d[PS] |E_{IB}(K_S) + E_{DE}(K_S)|^2}$$

$$\tilde{\epsilon}'_{+-\gamma} = \left[\hat{\epsilon} + \underbrace{\frac{i}{16} \frac{\epsilon'}{\epsilon} g_{E1}}_{< \epsilon'} e^{i\phi_\epsilon} \right] e^{i(\delta_1 - \delta_0 + \frac{\pi}{2})} \left| 2 \frac{E_\gamma}{M_K} \right|^2 (1 - \beta^2 \cos^2(\theta))$$

Final Results - $\eta_{+-\gamma}$

- Computed value of $\eta_{+-\gamma}$ is consistent with previous studies



Conclusions

- $K_{L,S} \rightarrow \pi^+ \pi^- \gamma$ data from 97 and 99 have been analyzed and amplitudes for direct emission have been measured
 - Most precise M1 parameters to date
 - Consistent with previous measurements
 - First non-zero measurement of E1 direct emission parameters
 - Indirect CP violating amplitude is non-zero at 1.7σ
 - Consistent with upper limit from $K_L \rightarrow \pi^+ \pi^- \varepsilon^+ \varepsilon^-$
 - Direct CP violating amplitude is non-zero at 3.1σ
 - Calculated value of $\eta_{+\gamma}$ is consistent with all previous values
 - Majority of E1 DE amplitude violates CP directly

Extra Slides

Theory Predictions

- My Result: $\frac{|\hat{\epsilon}|}{|\epsilon|} \approx 1.7$
- Only theory prediction (Tandean & Valencia, PRD 62 116007, 2000):
 - Standard Model: $\frac{|\hat{\epsilon}|}{|\epsilon|} < 3 \times 10^{-3}$
 - Estimates using short distance $s \rightarrow d\gamma$, $d\gamma$ transitions, left-right symmetric models and supersymmetric models produce limits of the same order of magnitude

Matrix Element for $K \rightarrow \pi^+ \pi^- \gamma$

- The effective matrix element for a general neutral kaon decay into $\pi^+(p_+) \pi^-(p_-) \gamma(q, \epsilon)$ is

$$M = -\frac{e M_{K \rightarrow \pi^+ \pi^-}}{(M_K)^4} \left[[E_{IB}(K) + E_{DE}(K)] * \left[(\epsilon \cdot p_-)(q \cdot p_+) - (\epsilon \cdot p_+)(q \cdot p_-) \right] \right. \\ \left. + M_{DE}(K) \left[\epsilon_{\lambda \mu \rho \sigma} \epsilon^\lambda p_+^\mu p_-^\rho q^\sigma \right] \right]$$

where

E_{IB} is the amplitude for the Inner Bremsstrahlung transition

E_{DE} is the amplitude for a direct emission electric transition

M_{DE} is the amplitude for a direct emission magnetic transition

Decay Amplitudes

$$E_{IB}(K_S) = \left| 4 \frac{M_K^2}{E_\gamma^2} \right| \frac{e^{i\delta_0}}{1 - \beta^2 \cos^2(\theta)}$$

CP conserving

$$E_{IB}(K_L) = \left| 4 \frac{M_K^2}{E_\gamma^2} \right| \frac{\overbrace{\eta_{+-}}^{\epsilon + \epsilon'} e^{i\delta_0}}{1 - \beta^2 \cos^2(\theta)}$$

CP violating

$$M(K_S) = i \epsilon g_{M1} \left| \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2 E_\gamma M_K} + 1 \right| e^{i\delta_1}$$

CP violating

$$M(K_L) = i g_{M1} \left| \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2 E_\gamma M_K} + 1 \right| e^{i\delta_1}$$

CP conserving

$$E_{DE}(K_S) = \frac{g_{E1}}{\epsilon} e^{i(\delta_1 + \phi_\epsilon)}$$

CP conserving

$$E_{DE}(K_L) = \underbrace{g_{E1} e^{i(\delta_1 + \phi_\epsilon)}}_{\text{indirect CPV}} + i \underbrace{16 \hat{\epsilon} e^{i\delta_1}}_{\text{direct CPV}}$$

CP violating

Decay Amplitudes

$$E_{IB}(K_S) = \left| 4 \frac{M_K^2}{E_\gamma^2} \right| \frac{e^{i\delta_0}}{1 - \beta^2 \cos^2(\theta)}$$

Dominant for K_S

$$E_{IB}(K_L) = \left| 4 \frac{M_K^2}{E_\gamma^2} \right| \frac{\overbrace{\eta_{+-}}^{\epsilon + \epsilon'} e^{i\delta_0}}{1 - \beta^2 \cos^2(\theta)}$$

$$M(K_S) = i \epsilon g_{M1} \left| \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2 E_\gamma M_K} + 1 \right| e^{i\delta_1}$$

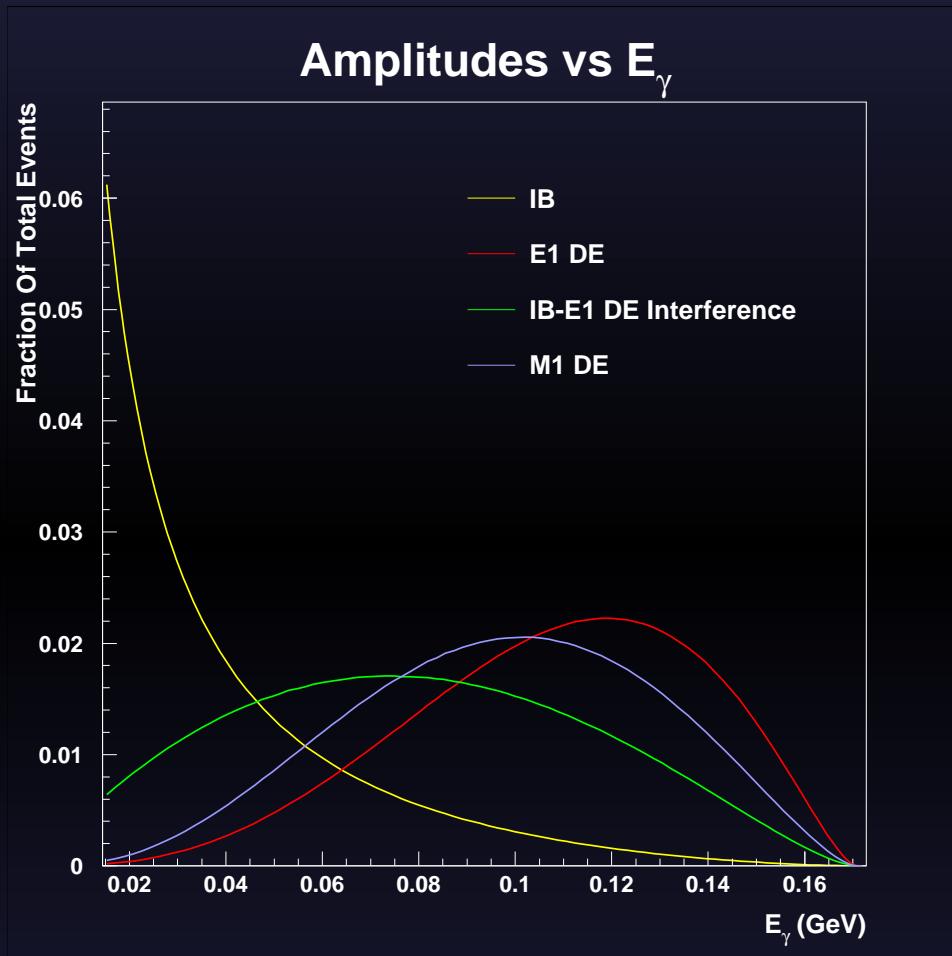
$$M(K_L) = i g_{M1} \left| \frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2 E_\gamma M_K} + 1 \right| e^{i\delta_1}$$

Dominant for K_L

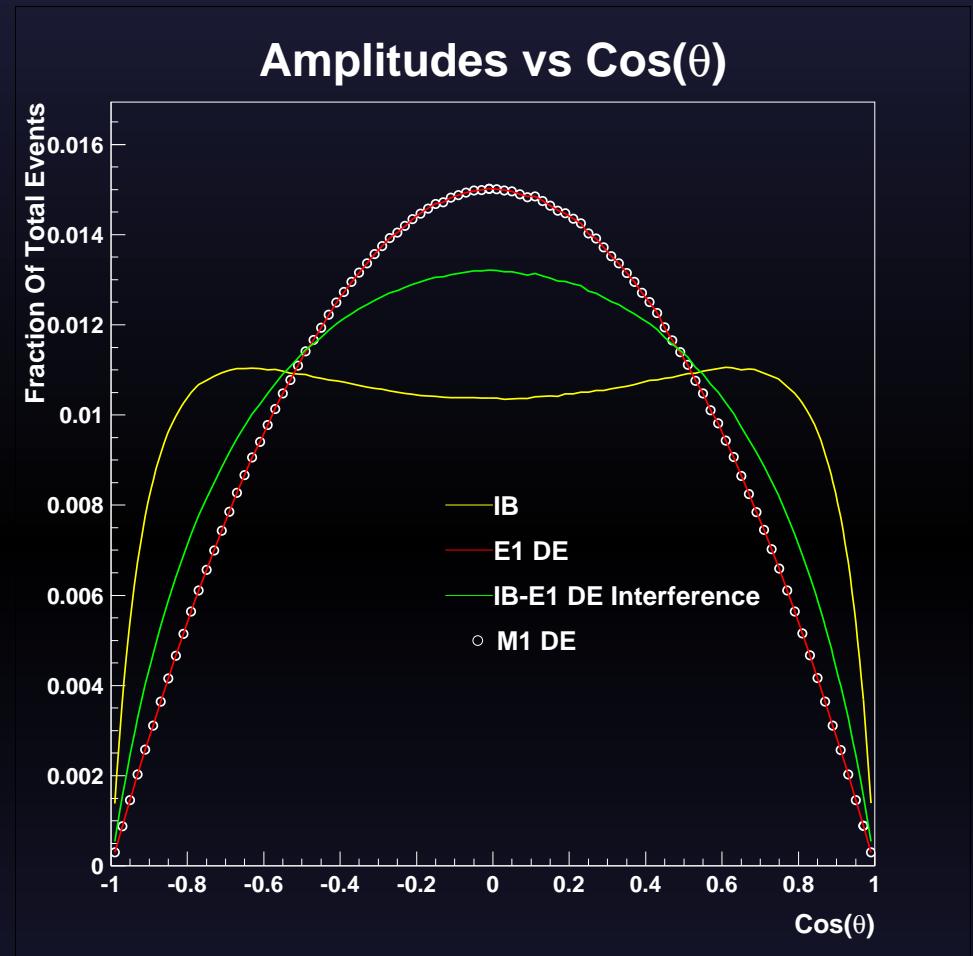
$$E_{DE}(K_S) = \frac{g_{E1}}{\epsilon} e^{i(\delta_1 + \phi_\epsilon)}$$

$$E_{DE}(K_L) = \underbrace{g_{E1} e^{i(\delta_1 + \phi_\epsilon)}}_{\text{indirect CPV}} + \underbrace{i 16 \hat{\epsilon} e^{i\delta_1}}_{\text{direct CPV}}$$

Amplitudes



Dependence On E_γ



Dependence On $\cos\theta$

Starting point for decay rate

- We start by assuming that we have a state of the form

$$|\Psi\rangle = |K_L\rangle + \rho |K_S\rangle$$

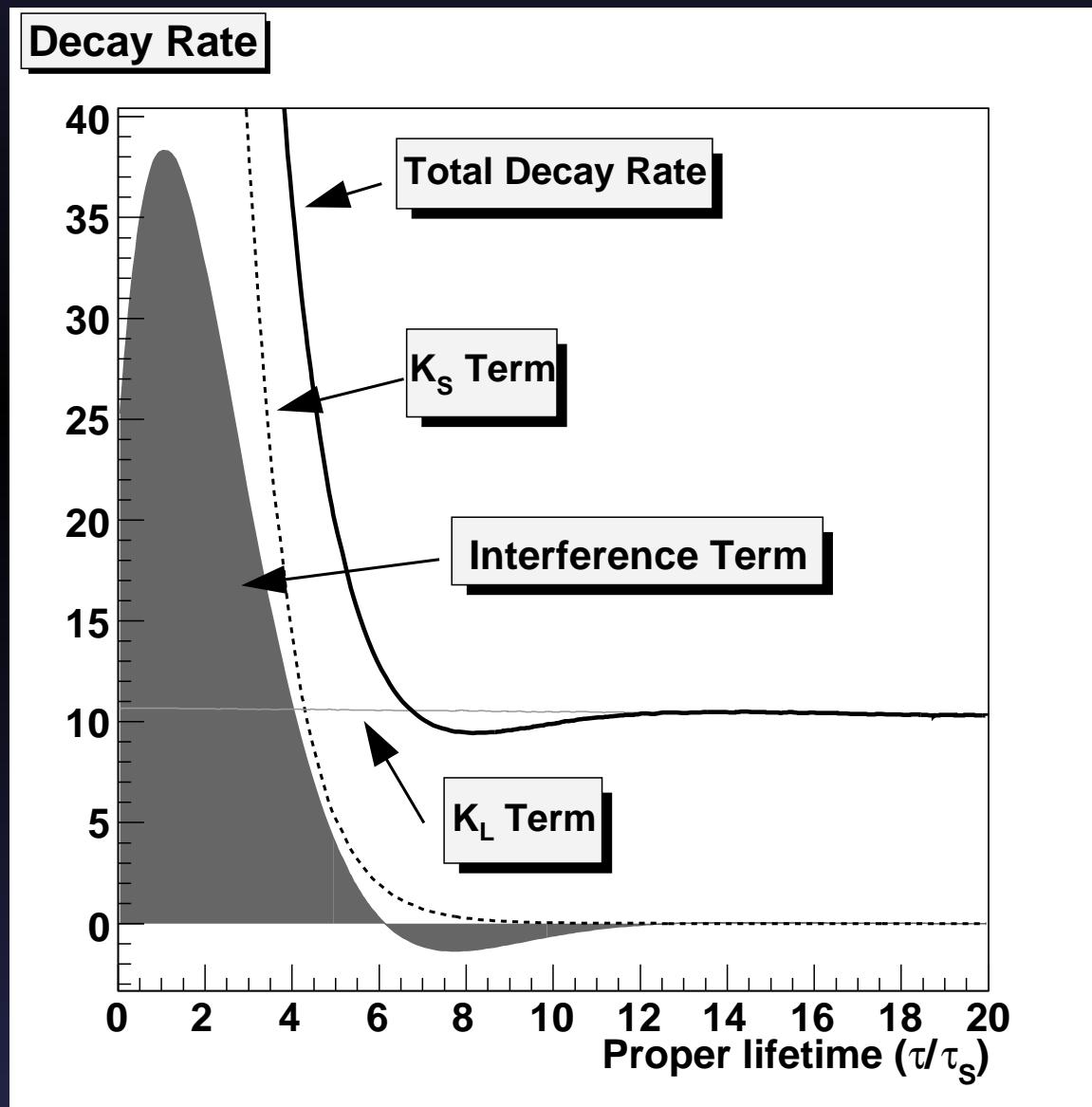
ρ controls the size of the K_S component relative to the total

- Passing a beam of K_L through material will transform part of the beam to K_S . ρ describes the strength of this effect, and is called the regeneration amplitude
- Then, using this state, we begin to calculate the decay rate of the beam.
 - Squared matrix element:

$$|M_{total}|^2 = |M_L + \rho M_S|^2$$

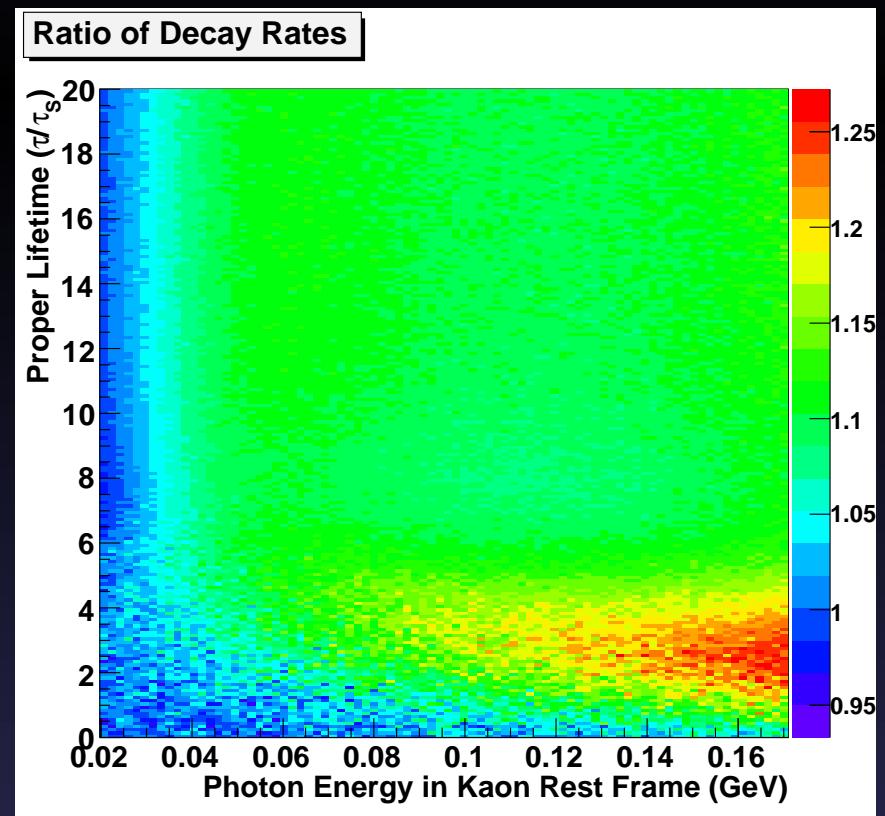
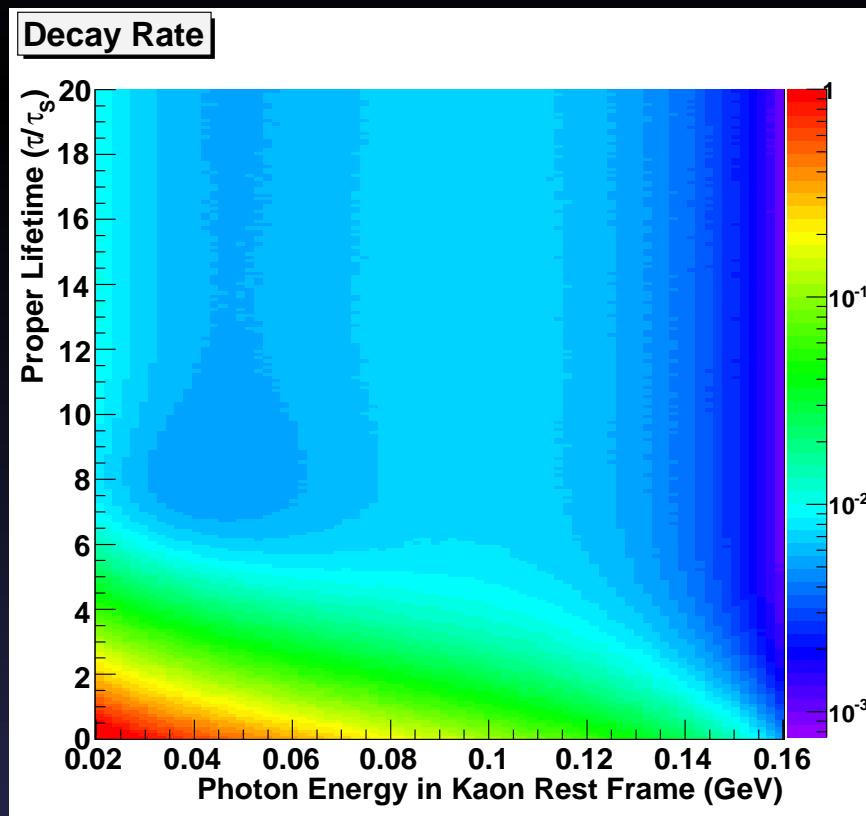
Appearance of Decay Rate

The time-dependent decay rate is composed of three components including the interference term



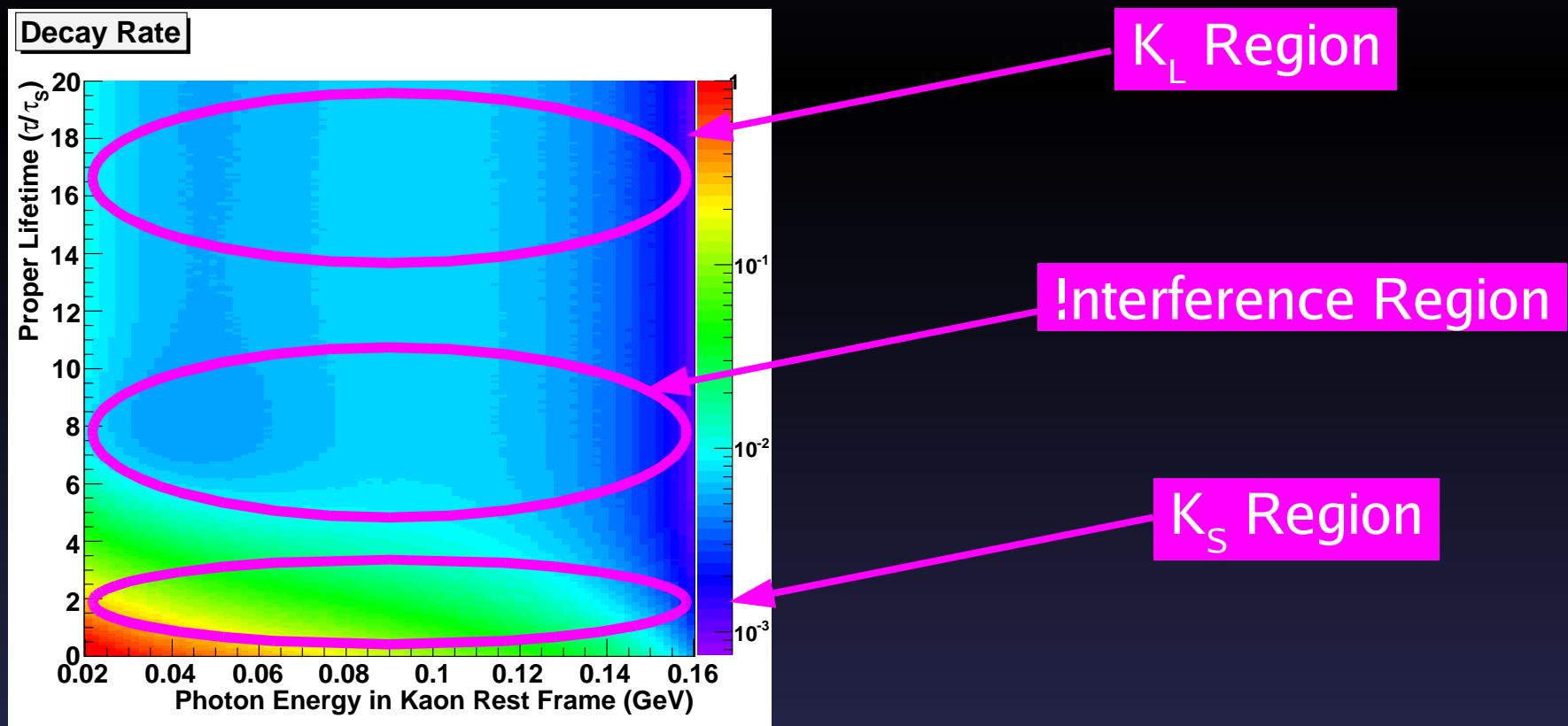
Projections of Decay Rate

- The decay rate will give the density of events in phase space (τ , E_γ , $\cos\theta$)
- Plot of photon energy versus proper lifetime is interesting:



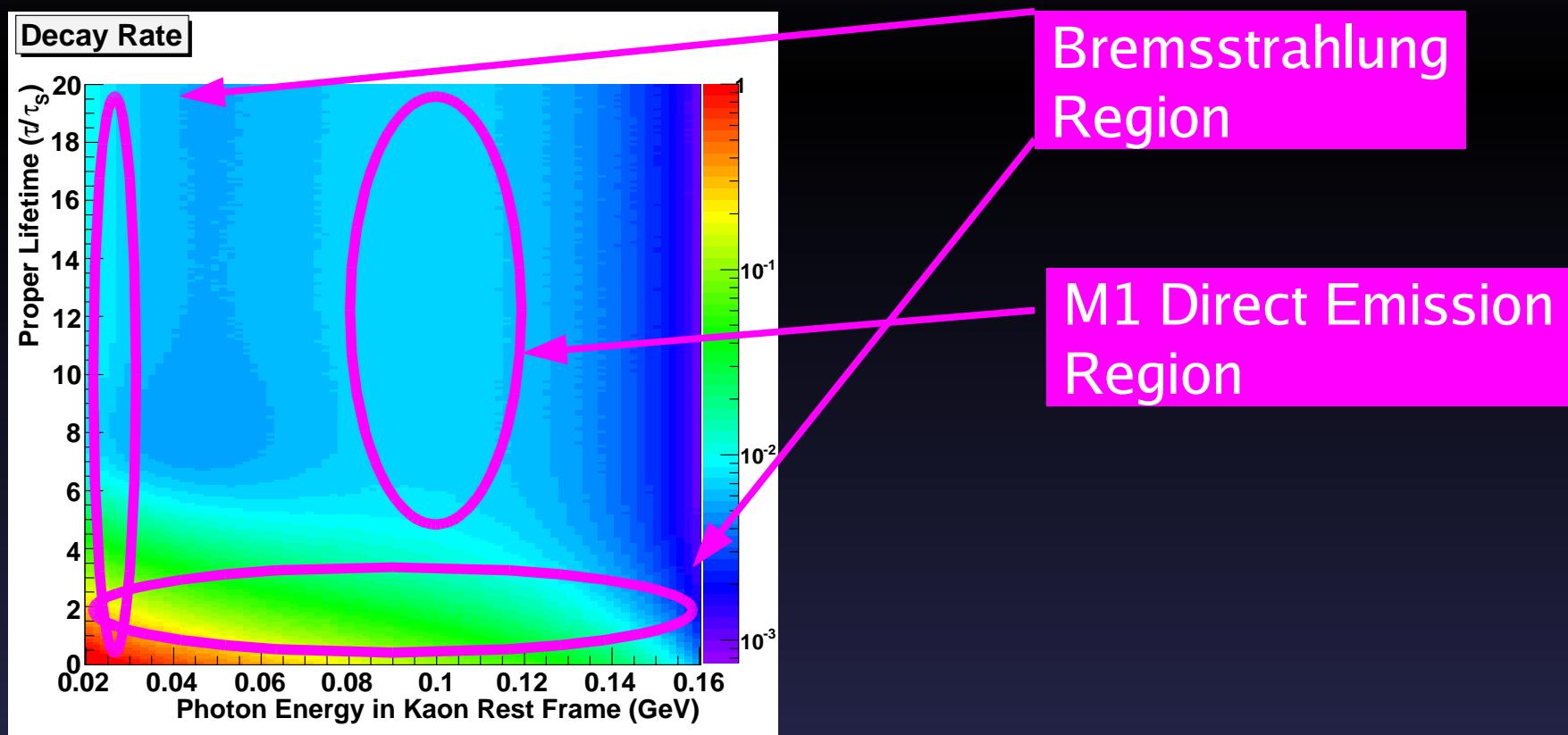
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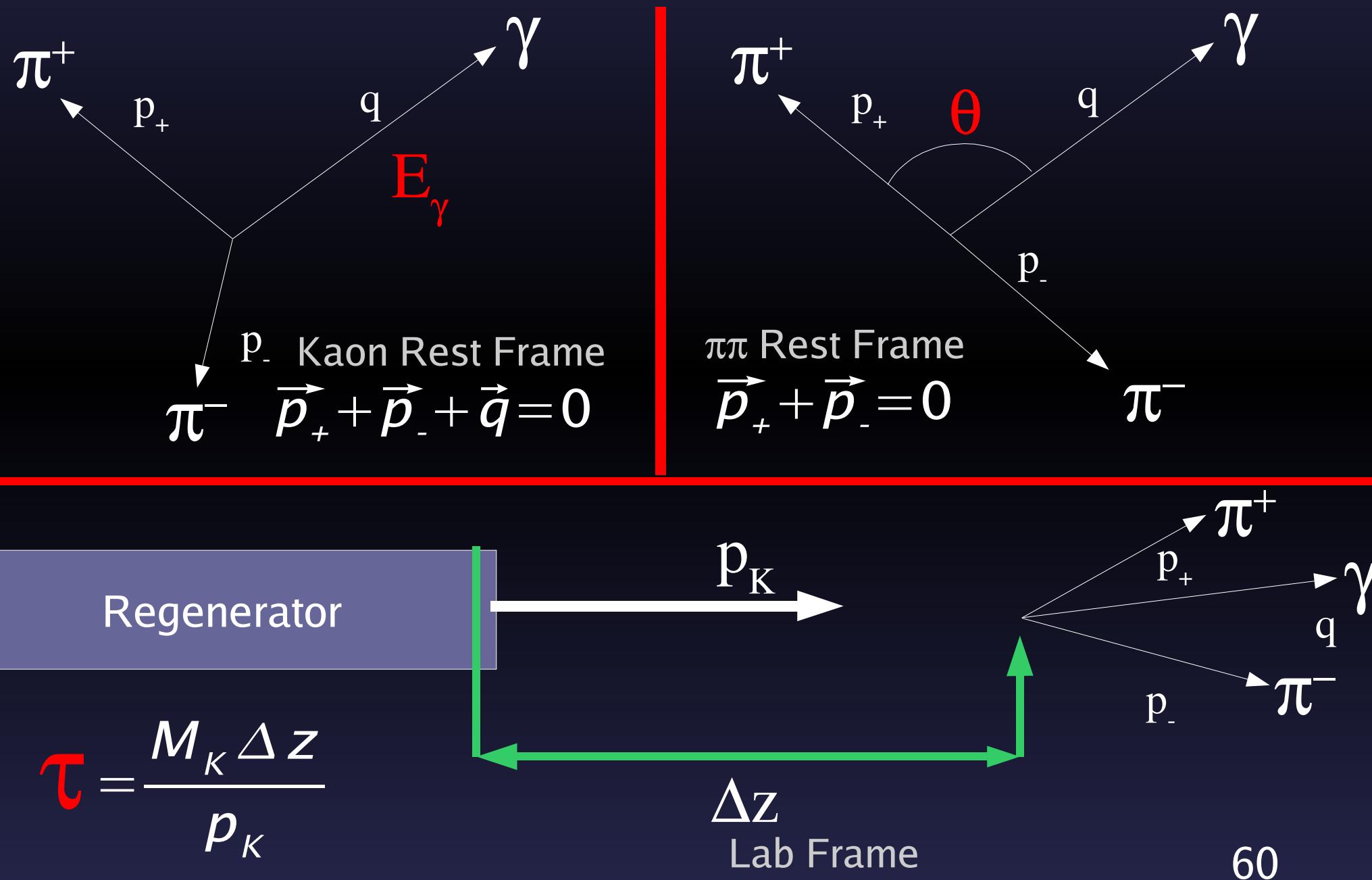


Projections of Decay Rate

- The decay rate will give the density of events in phase space (τ , E_γ , $\cos\theta$)
- Plot of photon energy versus proper lifetime is interesting:



Kinematic Variables for $K \rightarrow \pi^+ \pi^- \gamma$

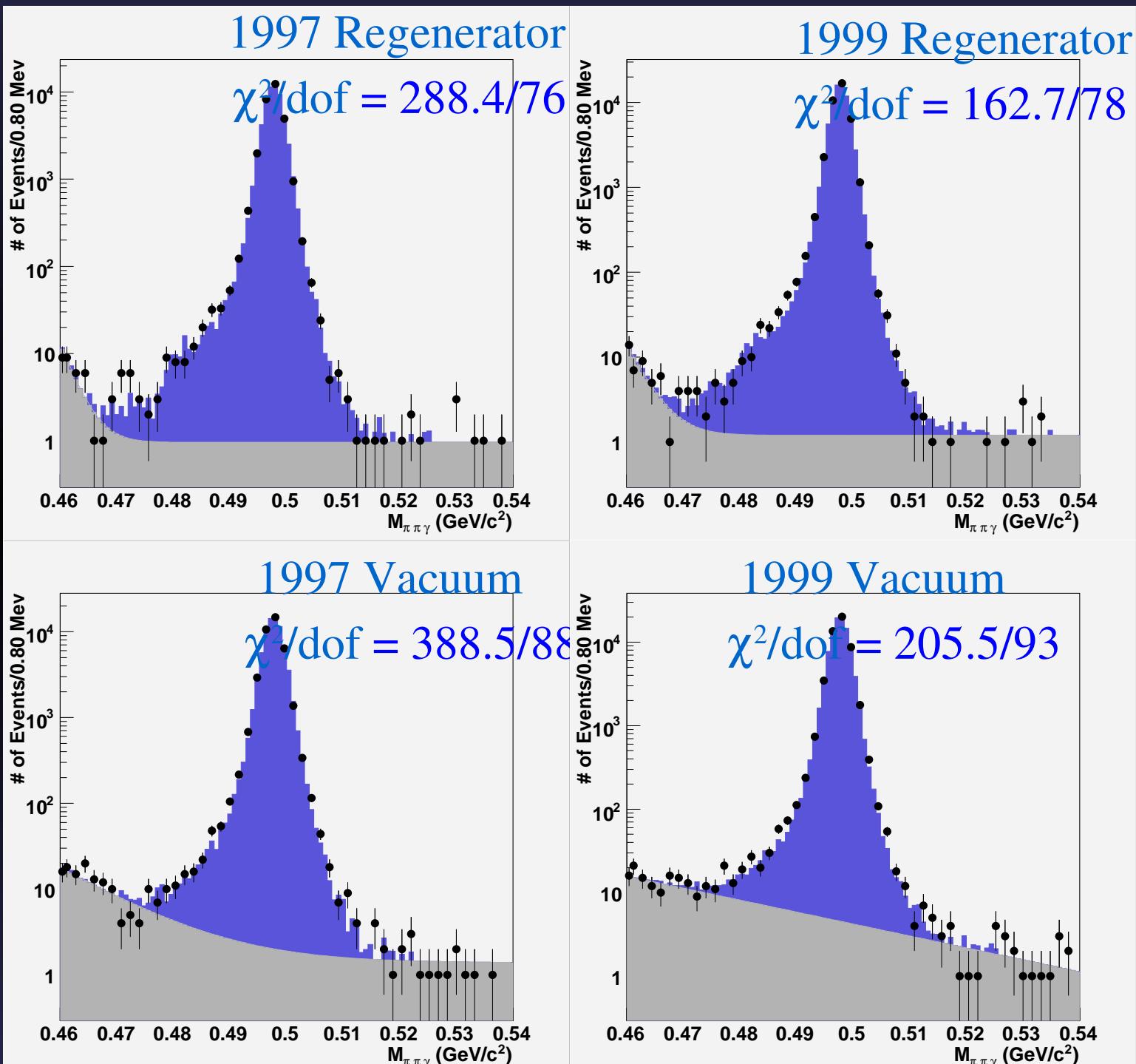


Analysis Cuts

Cut Variable	Keep Event If...
Kaon Mass	$0.48967 \text{ GeV}/c^2 < M_{\pi^+\pi^-\gamma} < 0.50567 \text{ GeV}/c^2$
P_T^2 w.r.t Regenerator	$P_T^2 < 2.5 \times 10^{-4} \text{ GeV}^2/c^2$
Kaon Momentum	$40.0 \text{ GeV}/c < P_{\pi^+\pi^-\gamma} < 160.0 \text{ GeV}/c$
Photon Energy in Lab Frame	$E_\gamma^* > 1.5 \text{ GeV}$
Photon Energy in Kaon Rest Frame, From Calorimeter	$20.0 \text{ MeV} < E_\gamma^* < 175.0 \text{ MeV}$
Photon Energy in Kaon Rest Frame, From Kinematics	$20.0 \text{ MeV} < E_\gamma^* < 175.0 \text{ MeV}$
$\pi\pi$ Invariant Mass, Implied From Above Cut	$0.2711 \text{ GeV}/c^2 < M_{\pi\pi} < 0.4772 \text{ GeV}/c^2$
Shape χ^2 For Photon Cluster	$\chi^2 < 48$
Outer Fiducial Cut For Photon Cluster	$\text{ISEEDRING} < 18.1 \text{ cm}$
Inner Fiducial Cut For Photon Cluster	$\text{ISMLRING2} > 4.5 \text{ cm}$
Photon/Track Separation at CsI	$d > 30 \text{ cm}$
Number of CsI clusters	$\text{NCLUS} \geq 3$
pp0kin w.r.t. Target	$-0.10 \text{ GeV}^2/c^2 < P_{\pi^0}^2 < -0.0055 \text{ GeV}^2/c^2$ passes
L3 pp0kin	
Z vertex	$125.5 \text{ m} < \text{VTXZ} < 158.0 \text{ m}$
E/p	$E/p < 0.85$
Track Momentum	$\text{TRKP} > 8.0 \text{ GeV}$
Vertex χ^2	$\text{VTXCHI} < 50.0$
Magnet Offset χ^2	$\text{TRKOCHI} < 50.0$
Track x separation at CsI	$\Delta x > 3.0 \text{ cm}$
Track y separation at CsI	$\Delta y > 3.0 \text{ cm}$
Total track separation at CsI	$\Delta r > 20.0 \text{ cm}$
Number of Tracks	$\text{NTRK} = 2$
$\Lambda \rightarrow p\pi$ invariant mass	$M_{p\pi} < 1.112 \text{ GeV}/c^2$ or $M_{p\pi} > 1.119 \text{ GeV}/c^2$
Early energy in photon cluster	$\text{ADCSI_EARLY} < 150 \text{ counts}$
In-time energy in photon cluster	$\text{ADCSI_INTIM} > 115 \text{ counts}$
Photon/Upstream Track Projection at CsI	$d > 2.0 \text{ cm}$ distance
Reconstruction Routines	Return no errors
Veto Cuts	All pass
Level 1 Trigger Verification	Event passes
Fiducial Cuts	All pass
Number of Photon Candidates That Pass ALL Cuts	$N_{COMBINATIONS} = 1$ ONLY

Data sample

- After all cuts there are ~307,000 events in the total sample
- See list of cuts in back of talk
- <244 background events (0.08%)
- 40% $K \rightarrow \pi\mu\nu$
- 30% $K \rightarrow \pi e\nu$
- 30% $K \rightarrow \pi^+\pi^-\pi^0$



Likelihood function

Normalization
Monte Carlo

$$\log \mathcal{L}(\vec{\alpha}) = \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) \\ + \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) \\ - (N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG})$$

data

$$\times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} \right. \\ \left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} \right]$$

Likelihood function

$x_i = (E_\gamma, \cos\theta, z, p_K)$ decay rate

$$\log \mathcal{L}(\vec{\alpha}) = \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha})$$

$$+ \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) \quad \text{fit parameters}$$

$$- (N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG})$$

$$\times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} \right]$$

$$+ N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} \right]$$

Likelihood function

$$\begin{aligned}
 \log \mathcal{L}(\vec{\alpha}) = & \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) \\
 & + \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i; \vec{\alpha}) \\
 & - (N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}) \\
 & \times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} \right. \\
 & \left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha})}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i; \vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i; \vec{\alpha}_0)}} \right]
 \end{aligned}$$

(E_γ, cosθ, z, p_K) decay rate
fit parameters initial guess parameters
generation parameters

Propagation of correlated errors

- Each group of correlated errors can be combined using:

$$U_{ii} = \sum_{k,l} \frac{\partial \phi_i}{\partial \theta_l} \frac{\partial \phi_i}{\partial \theta_k} V_{kl}$$

Total Variance of fit parameter i

Shift in fit parameter i when input parameter l is varied within uncertainty

Sum over correlated input parameters

Covariance between input parameters k and l

Final Results - $\eta_{+-\gamma}$

- Computed value of $\eta_{+-\gamma}$ is consistent with previous studies

